

CSE548/AMS542 Fall 2008 Analysis of Algorithms

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Due **September 18th** before class. Each problem, unless specified otherwise, has a maximum of 10 points. (i) You write down the solution clearly. If we can not recognize your writing then you may lose points. (ii) Avoid too many details. A succinct and clean proof is the best. You may use the algorithms we covered in class without referring to the details. (iii) If you discuss some of the problems with another fellow student (at most 2 students per group), write down his/her name and the problems. If you consult any books/webpages, cite them.

Homework 1

1. Prove or disprove (i.e., give counter examples) for the following claims. $f(n), g(n)$ are non-negative functions.

- (a) $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.
- (b) $o(f(n)) \cap \omega(f(n)) = \emptyset$.
- (c) $(n + a)^b = \Theta(n^b)$, a, b are positive integers.
- (d) $f(n) = O(f(n)^2)$.
- (e) $f(n) = O(g(n))$ implies that $2^{f(n)} = O(2^{g(n)})$.

2. Prove claim (3.2) on page 78.

Let G be an undirected graph on n nodes. Prove that any two of the following statements implies the third.

- (a) G is connected;
 - (b) G does not contain a cycle;
 - (c) G has $n - 1$ edges.
3. The diameter of an unweighted graph $G = (V, E)$ is the largest of the all shortest-path distances in the graph.
 - (a) Give an algorithm to calculate the diameter of a graph. Analyze its running time.
 - (b) Give an $O(n + m)$ algorithm to *estimate* the diameter of a graph up to an approximation factor of 2. That is, calculate an approximate diameter d such that the true diameter is at least d and at most $2d$.

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4. A directed graph is semi-connected if for all pairs of vertices u, v , we have either a path from u to v or a path from v to u . Give an efficient algorithm to determine whether or not G is semi-connected. Also analyze the algorithm's running time.
5. How can the number of strongly connected components of a graph change when an edge is added between two existing vertices. Give examples for each possible case and prove those are the only possible cases.