CSE548/AMS542 Spring 2008 Analysis of Algorithms

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February 13, 2008

Due **Feb 26th** before class. Each problem, unless specified otherwise, has a maximum of 10 points. (i) You write down the solution clearly. If we can not recognize your writing then you may lose points. (ii) Avoid too many details. A succinct and clean proof is the best. You may use the algorithms we covered in class without referring to the details. (iii) If you discuss some of the problems with another fellow student (at most 2 students per group), write down his/her name and the problems. If you consult any books/webpages, cite them.

Homework 2

1. Consider the following problem. The input is a collection $A = \{a_1, \cdots, a_n\}$ of n points on the real line. The problem is to find a minimum cardinality collection S of unit intervals that cover every point in A. Another way to think about this same problem is the following. You know a collection of times (A) that trains will arrive at a station. When a train arrives there must be someone manning the station. Due to union rules, each employee can work at most one hour at the station. The problem is to find a scheduling of employees that covers all the times in A and uses the fewest number of employees.
   (a) Prove or disprove that the following algorithm correctly solves this problem. Let I be the interval that covers the most number of points in A. Add I to the solution set S. Then recursively continue on the points in A not covered by I.
   (b) Prove or disprove that the following algorithm correctly solves this problem. Let a_j be the smallest (leftmost) point in A. Add the interval $I = (a_j, a_{j+1})$ to the solution set S. Then recursively continue on the points in A not covered by I.

2. Consider the following problem. The input consists of n skiers with heights $p_1, \cdots, p_n$, and n skies with heights $s_1, \cdots, s_n$. The problem is to assign each skier a ski to to minimize the average difference between the height of a skier and his/her assigned ski. That is, if the ith skier is given the $\alpha(i)$th ski, then you want to minimize

$$\sum_{i=1}^{n} |p_i - s_{\alpha(i)}|.$$

   (a) Consider the following greedy algorithm. Find the skier and ski whose height difference is minimized. Assign this skier this ski. Repeat the process until every skier has a ski. Prove of disprove that this algorithm is correct.

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(b) Consider the following greedy algorithm. Give the shortest skier the shortest ski, give the second shortest skier the second shortest ski, give the third shortest skier the third shortest ski, etc. Prove of disprove that this algorithm is correct.

3. The input to this problem consists of an ordered list of n words. The length of the ith word is w_i, that is the ith word takes up w_i spaces. (For simplicity assume that there are no spaces between words.) The goal is to break this ordered list of words into lines, this is called a layout. Note that you can not reorder the words. The length of a line is the sum of the lengths of the words on that line. The ideal line length is L. No line may be longer than L, although it may be shorter. The penalty for having a line of length K is $L - K$. There are two ways to define the total penalty as shown below. The problem is to find a layout that minimizes the total penalty.

   (a) In the first definition, the total penalty is the sum of the line penalties.

   (b) In the second definition, the total penalty is the maximum of the line penalties.

   Prove or disprove that the following algorithm gives the correct solution for each of the problems above.

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For i = 1 to n
      Place the ith word on the current line if it fits
            else place the ith word on a new line
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4. You wish to drive from point A to point B along a highway minimizing the time that you are stopped for gas. You are told beforehand the capacity C of you gas tank in liters, your rate F of fuel consumption in liters/kilometer, the rate r in liters/minute at which you can fill your tank at a gas station, and the locations $A = x_1, \cdots, B = x_n$ of the gas stations along the highway. So if you stop to fill your tank from 2 liters to 8 liters, you would have to stop for $6/r$ minutes. Consider the following two algorithms:

   (a) Stop at every gas station, and fill the tank with just enough gas to make it to the next gas station.

   (b) Stop if and only if you don't have enough gas to make it to the next gas station, and if you stop, fill the tank up all the way.

   For each algorithm either prove or disprove that this algorithm correctly solves the problem. Your proof of correctness must use an exchange argument.

5. There are n cities and m trains between these cities. Each train i has a departure city $s(i)$ and the departure time $t_1(i)$, as well as a destination city $d(i)$ and the arrival time $t_2(i)$. Now given two cities A, B, we would like to find a train schedule from A to B so that one can arrive at B the earliest. You need to consider the following practical constraints: there might be multiple trains between the same pair of cities (with possibly different departure/arrival times though); the connecting train need to be at least 10 mins later than the arrival time of the previous train (to allow one to go from one platform to another). Find an algorithm of running time $O((m + n) \log n)$. (Hint: think about how to model this problem as a graph problem.)