CSE548/AMS542 Spring 2008 Analysis of Algorithms

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Due **April 22nd** before class. Each problem, unless specified otherwise, has a maximum of 10 points. (i) You write down the solution clearly. If we can not recognize your writing then you may lose points. (ii) Avoid too many details. A succinct and clean proof is the best. You may use the algorithms we covered in class without referring to the details. (iii) If you discuss some of the problems with another fellow student (at most 2 students per group), write down his/her name and the problems. If you consult any books/webpages, cite them.

Homework 5

1. Prove claim (7.51) in textbook page 382.

2. You are given two sets A and B of points in the plane, where $n = |A| = |B|$. You are also given a real value r. Describe an $O(n^3)$ time algorithm that determines if there exists a one-to-one matching between the points of A to the points of B, and every point $a_i \in A$ is matched to a point $b_j \in B$ whose distance from a_i is at most r.

3. Given a directed graph $G = (V, E)$, each edge e has a positive capacity $c(e)$. Suppose the max flow f from a source s and a destination t is already computed. Now given the following modifications to the graph, show how to update the max flow as efficiently as possible (the algorithm should be significantly faster than computing the max flow from scratch). Each subproblem is given 5 points. The solution of computing the max flow from scratch does not get any points.

   (a) Increase the capacity of an edge e by 1.

   (b) Decrease the capacity of an edge e by 1.

4. Suppose Bob is using the max flow algorithm to solve for the maximum matching in a bipartite graph. However he made a mistake that he forgot to include the backward edges when he constructed the residual graph. Now he got a matching M. The question is, how large is M compared with the maximum matching? Give an example of a graph to show your bound is tight. (Note that you need to consider the asymptotic case, that is, your example should have n vertices with n possibly approaching infinity.)

5. You are given a set of n boxes, each specified by its height, width, and depth. The order of the dimensions is unimportant; for example, a $1 \times 2 \times 3$ box is exactly the same as a $3 \times 1 \times 2$ box or a $2 \times 1 \times 3$ box. You can nest box A inside box B if and only if A can be rotated so that it has strictly smaller height, strictly smaller width, and strictly smaller depth than B. Describe

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and analyze an efficient algorithm to nest all n boxes into as few groups as possible, where each group consists of a nested sequence. You are not allowed to put two boxes side-by-side inside a third box, even if they are small enough to fit.

6. (Extra credit problem) Let $G = (V, E)$ be a directed graph where the in-degree of each vertex is equal to its outdegree. Prove or disprove the following claim: For any two vertices u and v in G, the number of mutually edge-disjoint paths from u to v is equal to the number of mutually edge-disjoint paths from v to u.