CSE590 Fall 2009 Wireless and Mobile Networks

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Due **September 29th**. **Notice that we do not have lecture on 29th. Please submit your homework to the instructor mailbox in CS building.** Each problem, unless specified otherwise, has a maximum of 10 points. (i) You write down the solution clearly. If we can not recognize your writing then you may lose points. (ii) Avoid unnecessary details.

Homework 2

1. In class we covered Reed-Solomon code for an erasure channel, i.e., when symbols are completely erased and replaced by a question mark "?". If we apply Reed-Solomon code for a channel where bits are flipped, or symbols altered, we may use the following voting algorithm to decode. Use the same setting as in the lecture notes, suppose that the input data is x_0, x_1, x_{k-1} and the decoded data is $\beta_0, \beta_1, \cdots, \beta_{n-1}$. Now suppose we transmit the decoded data through a channel and the receiver receives the data with s errors. That is, s out of the n symbols received, $\beta'_0, \beta'_1, \cdots, \beta'_{n-1}$, are altered. We recover the original data by using all subsets of size k out of the n received symbols and check how many of these subsets recover the same solution. The solution that gets highest vote is supposed to be the correct answer.

   (a) How many votes will the correct solution get, at least?

   (b) How many votes will an incorrect solution get, at most?

   (c) Based on the previous two questions, what is the number of errors that can be tolerated by this correction algorithm?

2. The Hamming code (7, 4) takes a string of length 4 and outputs a codeword of length 7. This is by taking the multiplication of the input string with the encoding matrix:

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

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For example, an input string $x = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ will be encoded to

$$Hx = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

Here sum is in fact XOR (i.e., sum with mod 2). The question is, how many flip errors can be detected by this code?

3. To provide public telephone access to commercial ferries a telephone company installs a multi-channel wireless telephone system in a ferry. The wireless radio system connects to a base station on the shore through the air.

   (a) If the telephone company installs a four channel system, what is the probability of having a person come to the telephone and finding none of the lines are available? This probability is called the blockage probability. Assume that the average length of a telephone call is 3 minutes and each of the 150 passengers of the ferry make on the average one call per hour.

   (b) What is the average delay for accessing the telephones?

   (c) How many channels are needed to keep the blockage probability below 2 percent?