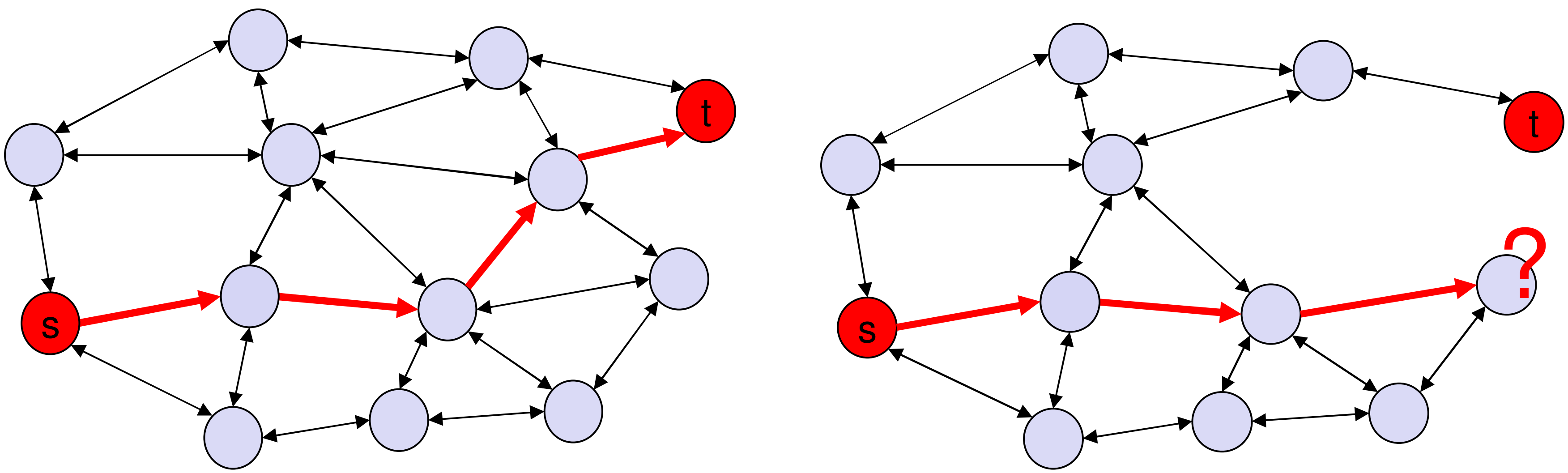
A brief overview of geographical routing

Geographical routing may get stuck

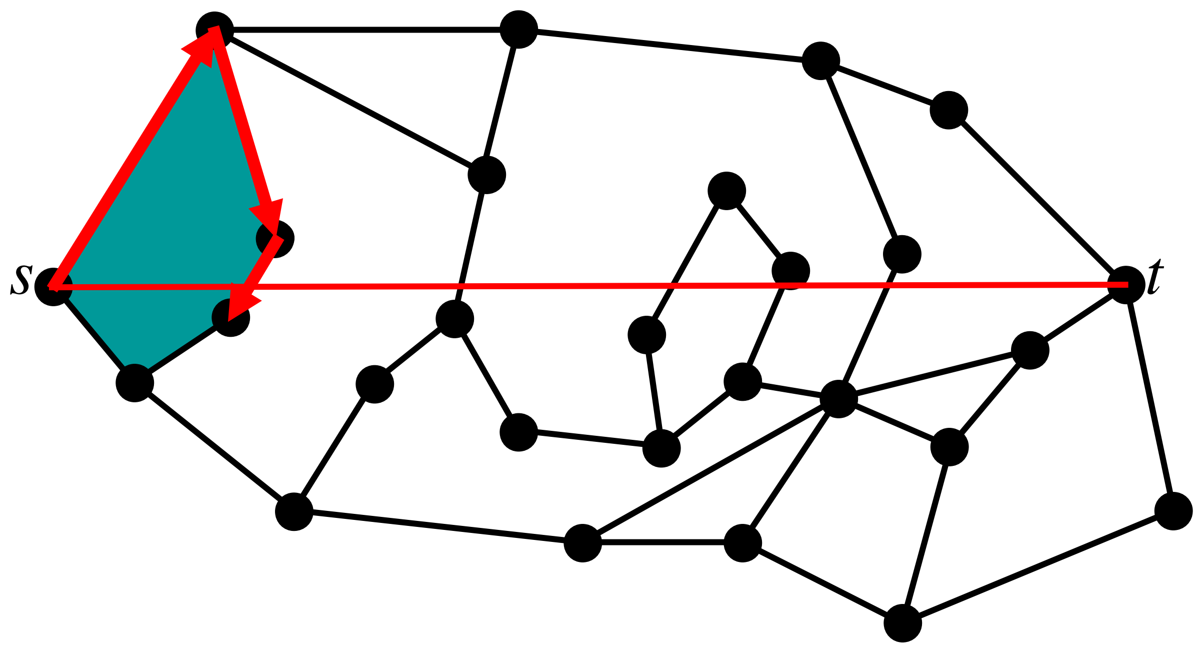
- Geographical routing may stuck at a node whose neighbors are all further away from the destination than itself.



Send packets to the neighbor closest to the destination

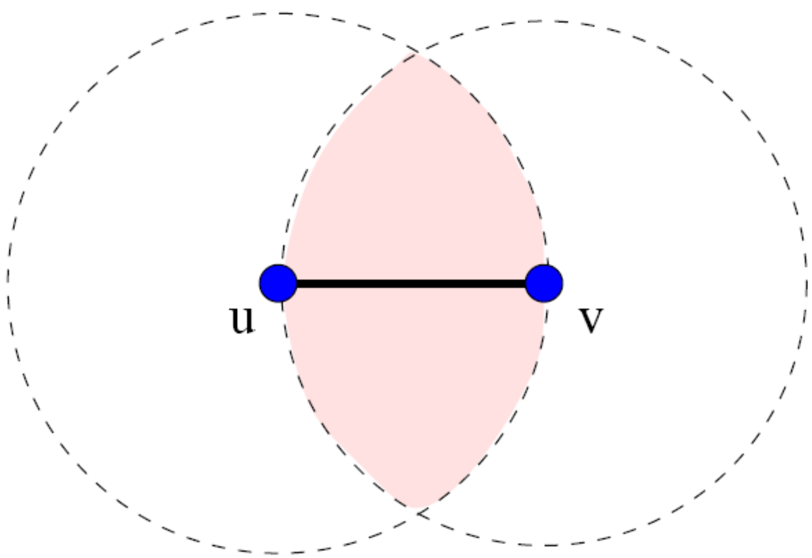
Face Routing

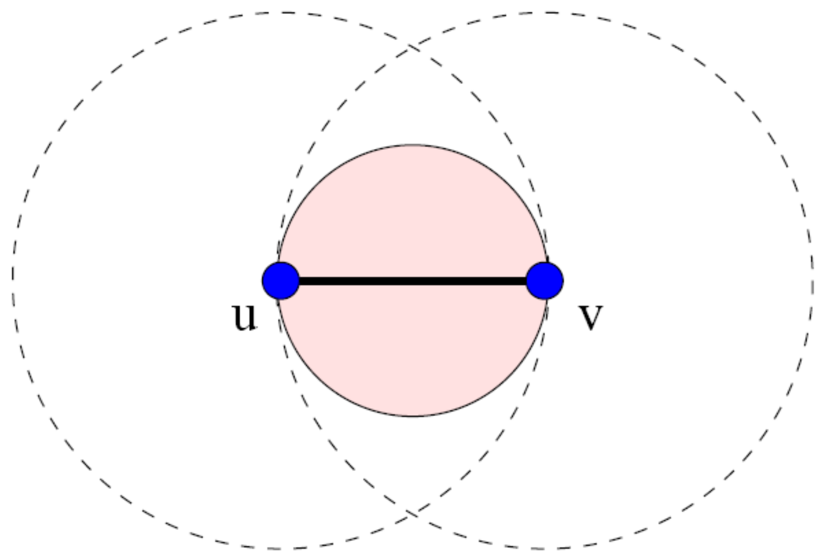
- Keep left hand on the wall, walk until hit the straight line connecting source to destination.
- Then switch to the next face.



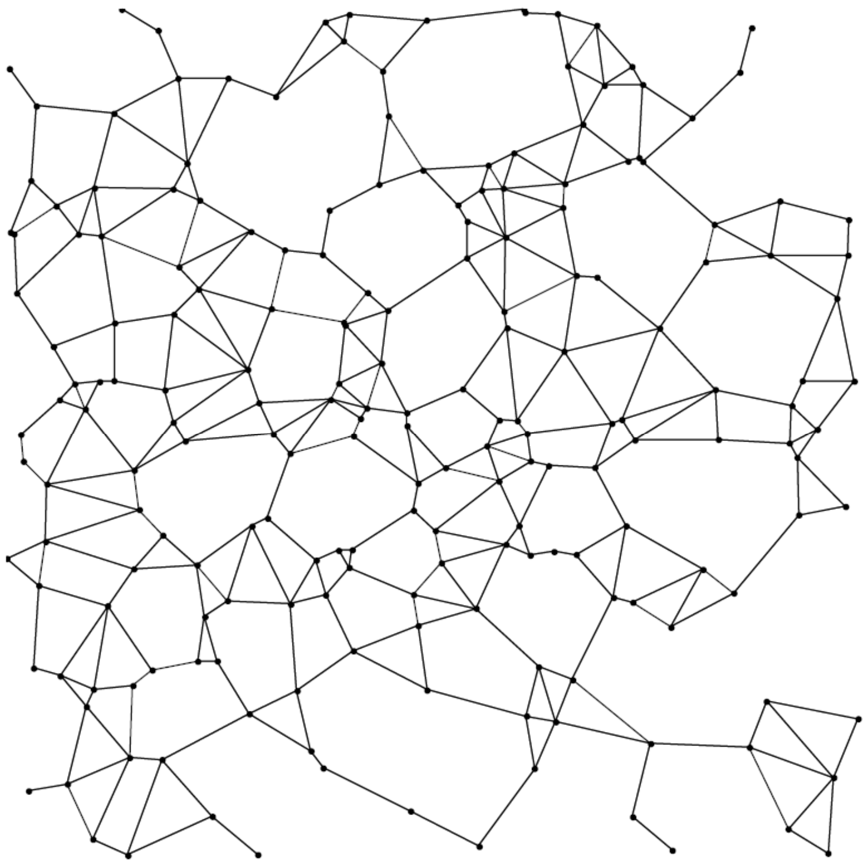
Relative Neighborhood Graph and Gabriel Graph

- Relative Neighborhood Graph (RNG) contains an edge uv if the lune is empty of other points.
- Gabriel Graph (GG) contains an edge uv if the disk with uv as diameter is empty of other points.
- Both can be constructed in a distributed way.

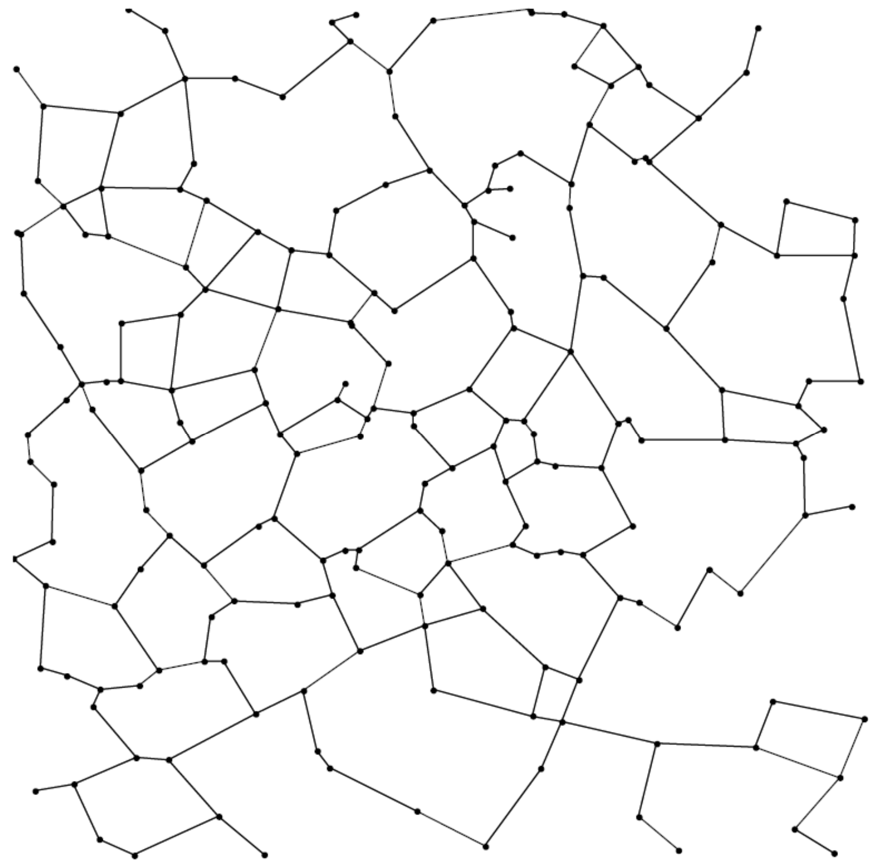




An example of GG and RNG



GG



RNG

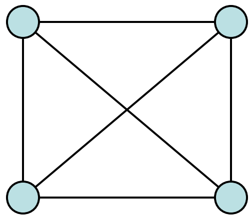
Two problems remain in geographical routing

- Both RNG and GG remove some edges → a short path may not exist!
- The shortest path on RNG or GG might be much longer than the shortest path on the original network.
- Even if the planar subgraph contains a short path, can greedy routing and face routing find a short one?

Tackle problem I:
Find a planar spanner

Find a good subgraph

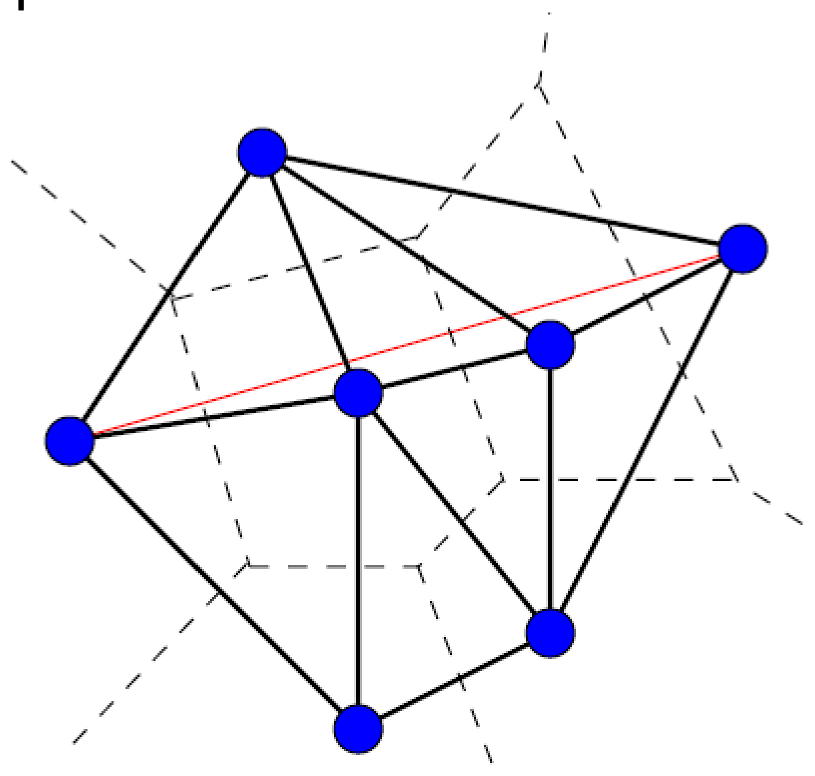
- Goal: a planar spanner such that the shortest path is at most α times the shortest path in the unit disk graph.
  - Euclidean spanner: The shortest path length is measured in total Euclidean length.
  - Hop spanner: The shortest path length is measured in hop count.
- α: spanning ratio.
  - Euclidean spanning ratio $\geq \sqrt{2}$
  - Hop spanning ratio ≥ 2.



- Let's first focus on Euclidean spanner.

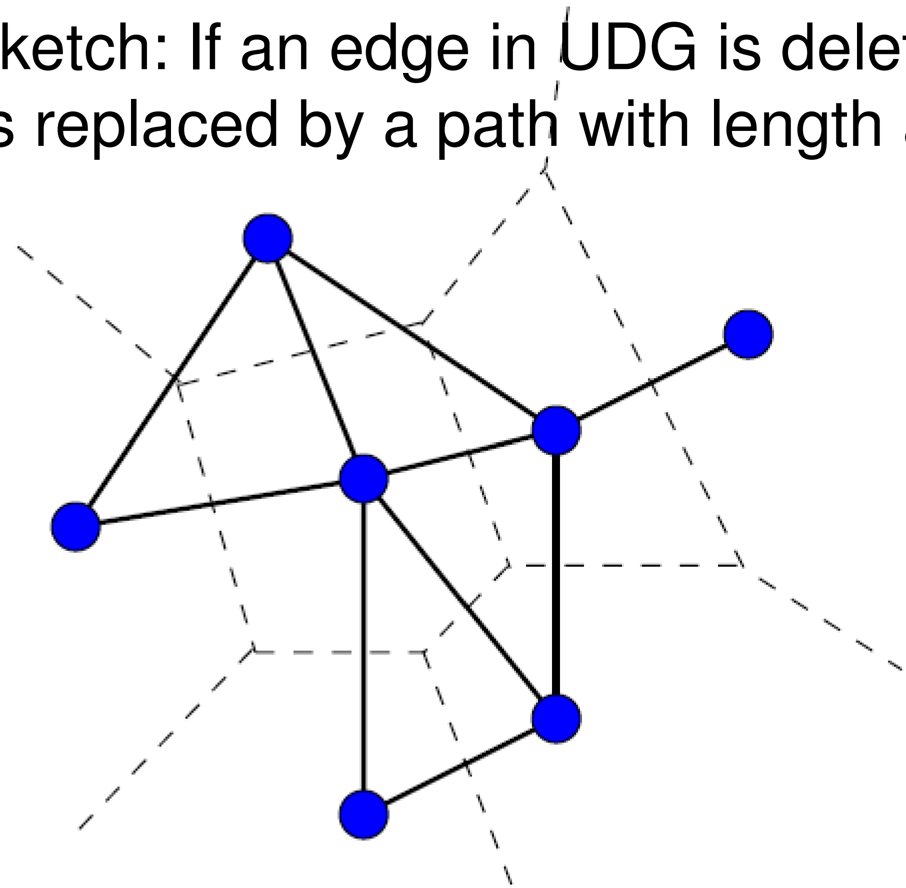
Delaunay triangulation is an Euclidean spanner

- DT is a 2.42-spanner of the Euclidean distance.
- For any two nodes uv, the Euclidean length of the shortest path in DT is at most 2.42 times |uv|.



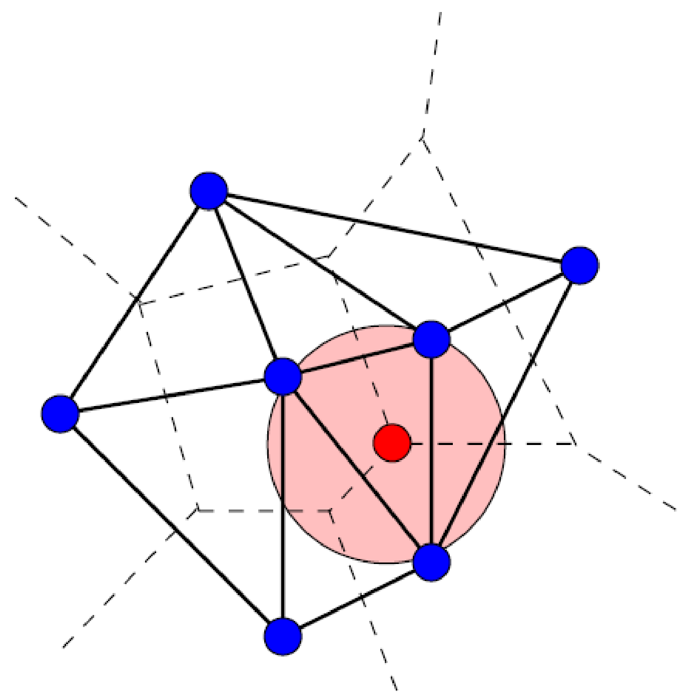
Restricted Delaunay graph

- Keep all the Delaunay edges no longer than 1.
- Claim: RDG is a 2.42-spanner (in total Euclidean length) of the UDG.
- Proof sketch: If an edge in UDG is deleted in RDG, then it's replaced by a path with length at most 2.42 longer.



Construction of RDG

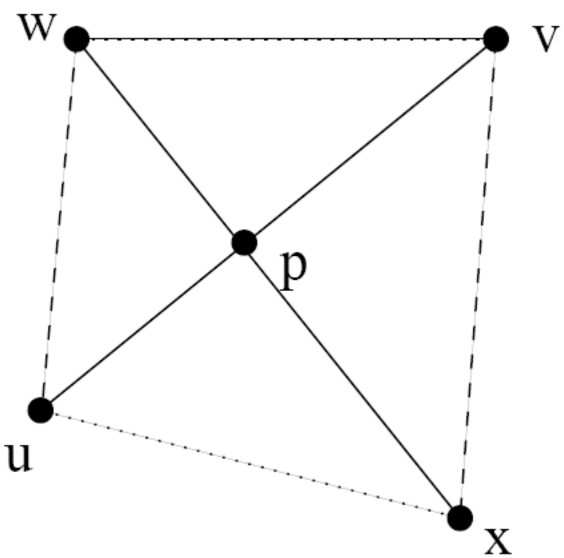
- Easy to compute a superset of RDG: Each node computes a local Delaunay of its 1-hop neighbors.
  - A global Delaunay edge is always a local Delaunay edge, due to the empty-circle property.
  - A local Delaunay may not be a global Delaunay edges.



- What if the superset has crossing edges?

Crossing Lemma

- Crossing lemma: if two edges cross in a UDG, then one node has edges to the three other nodes in UDG.



$|uw| \le |wp|+|up|$

$|vx| \le |vp|+|xp|$

➔ $|wu|+|vx| \le |wx|+|ux| \le 2$

Also, $|wv|+|ux| \le |wx|+|ux| \le 2$

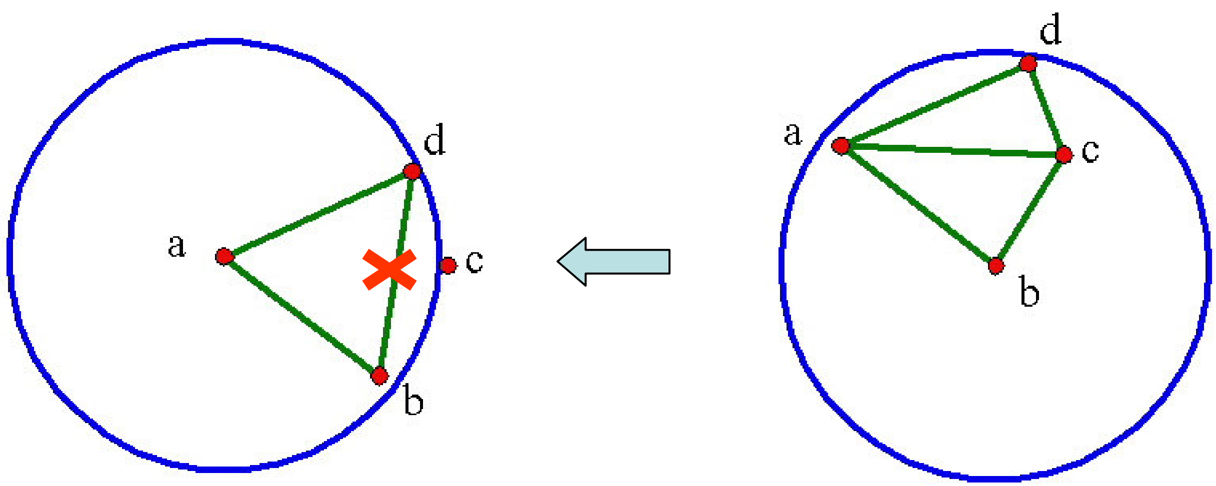
There must be 2 edges on the quad adjacent to the same node.

Detect crossings between local delaunay edges

- By the crossing Lemma: if two edges cross in a UDG, one of them has 3 nodes in its neighborhood and can tell which one is not Delaunay.
- Neighbors exchange their local DTs to resolve inconsistency.
  - A node tells its 1-hop neighbors the non-Delaunay edges in its local graph.
  - A node receiving a "forbidden" edge will delete it from its local graph.
- Completely distributed and local.

RDG construction

- 1-hop information exchange is sufficient.
  - Planar graph;
  - All the short Delaunay edges are included.
  - We may have some planar non-Delaunay edges but that does not hurt spanning property.



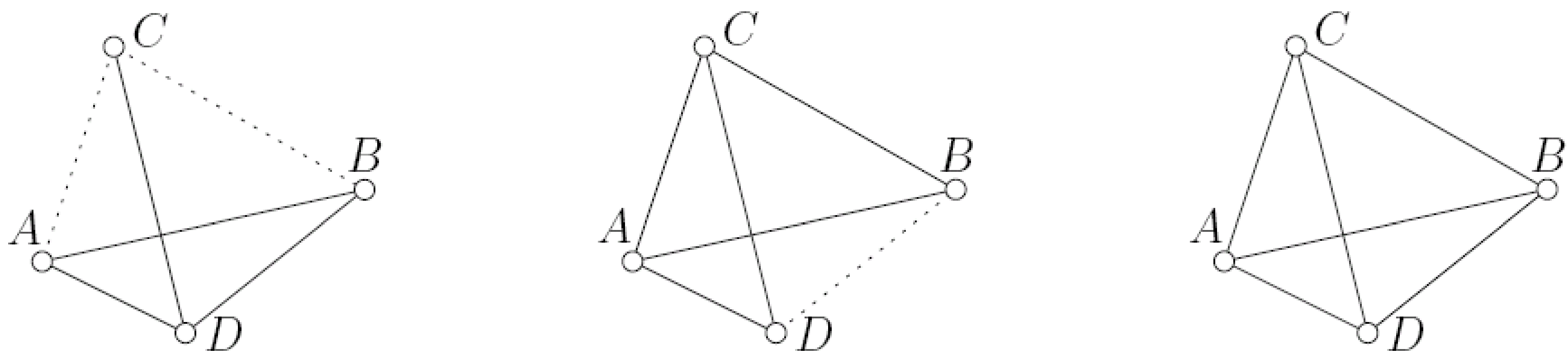
a's local Delaunay   b's local Delaunay

More on RDG construction

- RDG can be constructed without the full location information.
- Only local angle information suffices.
- Key operation: If two edges in the unit-disk graph cross, remove the one that is not in the Delaunay triangulation.
- How to tell that an edge is not in the Delaunay triangulation?

Removing non-Delaunay edges

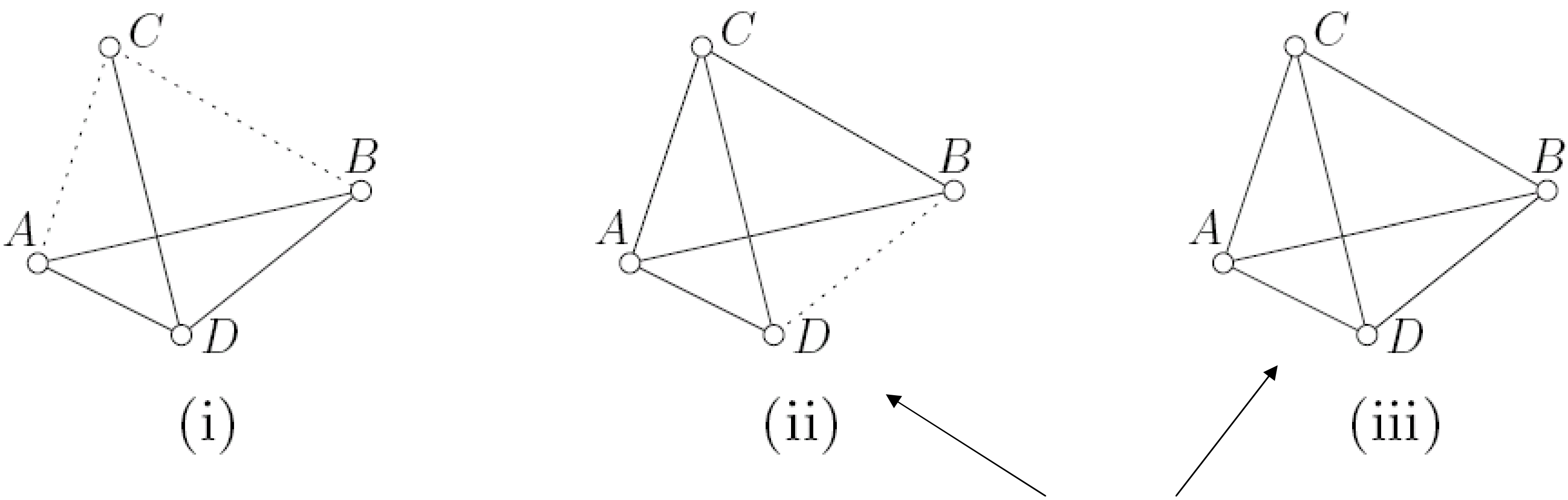
If two edges AB, CD cross, there are only three cases:



(i) (ii) (iii)

Removing non-Delaunay edges

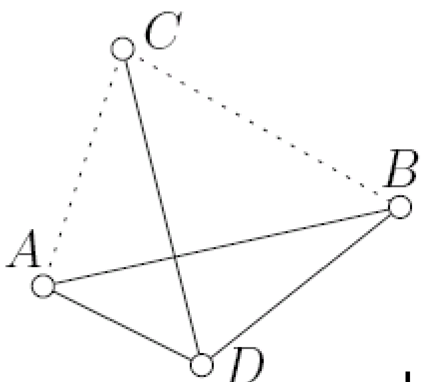
If two edges AB, CD cross, there are only three cases:



With angle info, the shape is fixed!
Node C can tell which edge is not Delaunay.

Removing non-Delaunay edges

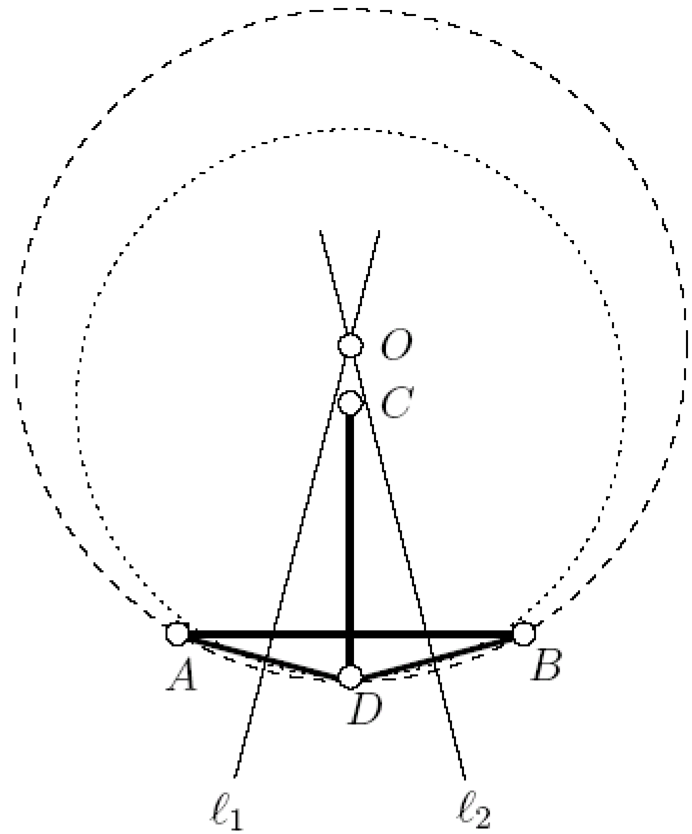
Case (i) : Use the "empty-circle" test of Delaunay triangulation



(i)

$|AC| > 1 \geq |CD|$

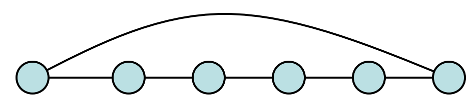
$|BC| > 1 \geq |CD|$



Conclusion: The edge AB is not a Delaunay edge.

Find a hop spanner

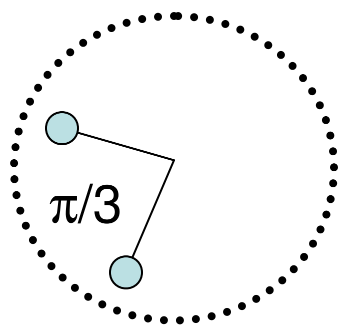
- Restricted Delaunay graph is not a hop spanner.
  - Take n nodes uniformly in a segment of length 1. The hop count can be as large as n-1.



- Reduce the density of the sensors.
  - Use clustering to reduce density.
  - Compute RDG on the subset to get a hop spanner.
  - Clustering also reduce interference and enables efficient resource reuse such as bandwidth.

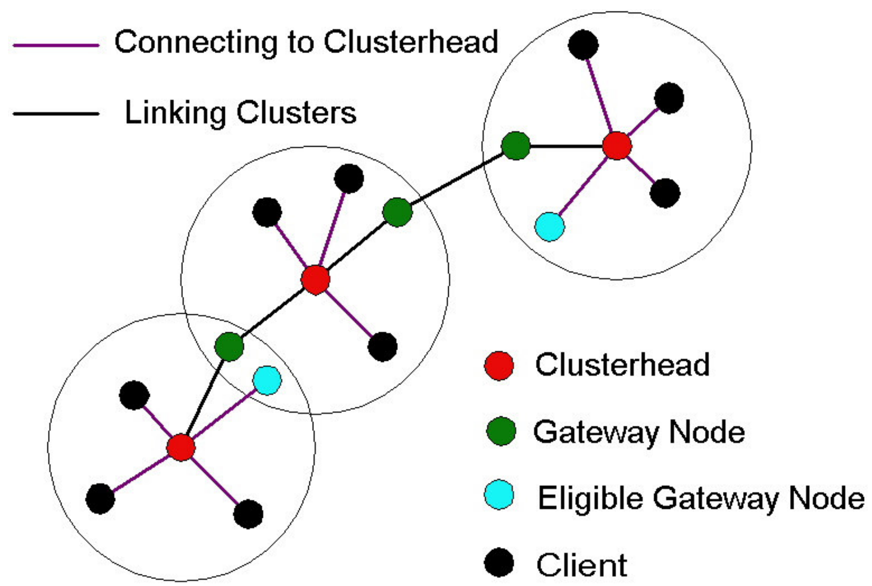
Reduce node density

- Find a subset of nodes, called clusterheads
  - Each node is directly connected to at least 1 clusterhead.
  - No two clusterheads are connected.
- Use a greedy algorithm. Pick a node as a clusterhead, remove all the 1-hop neighbors, continue.
- Constant density: ≤ 6 clusterheads in any unit disk.
  - The angle spanned by two clusterheads is at least $\pi/3$.



Connect clusterheads by gateways

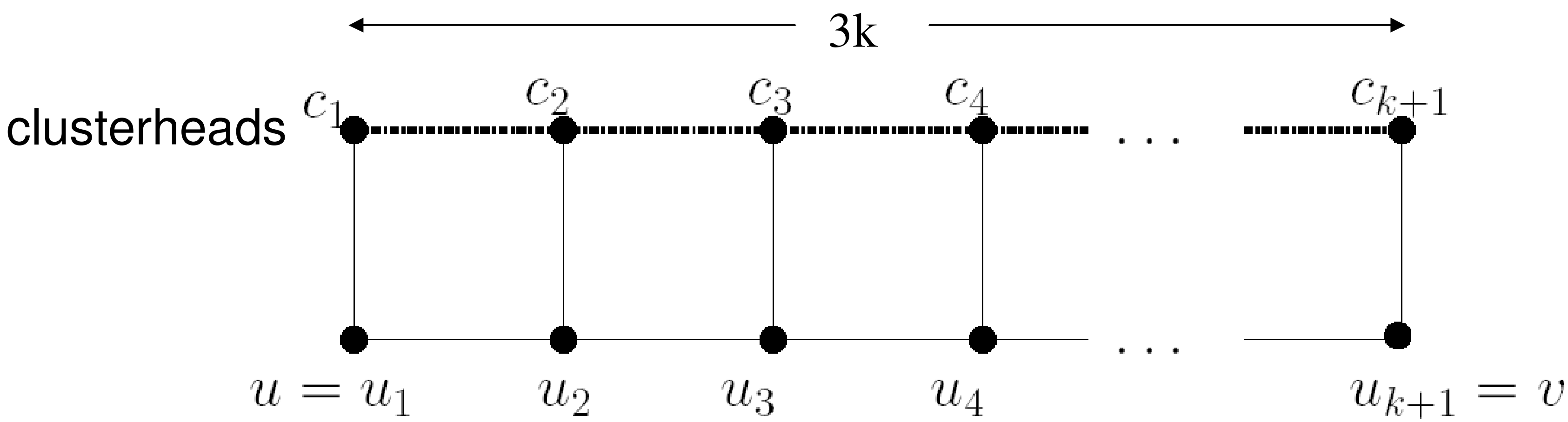
- For two clusterheads, if their clients have an edge, then we pick one pair as gateway nodes.



- Notice that clusterheads x, y are within 3 hops to have a pair of gateways.
- There are constant clusterheads and gateways inside any unit disk.

Path on clusterheads and gateways

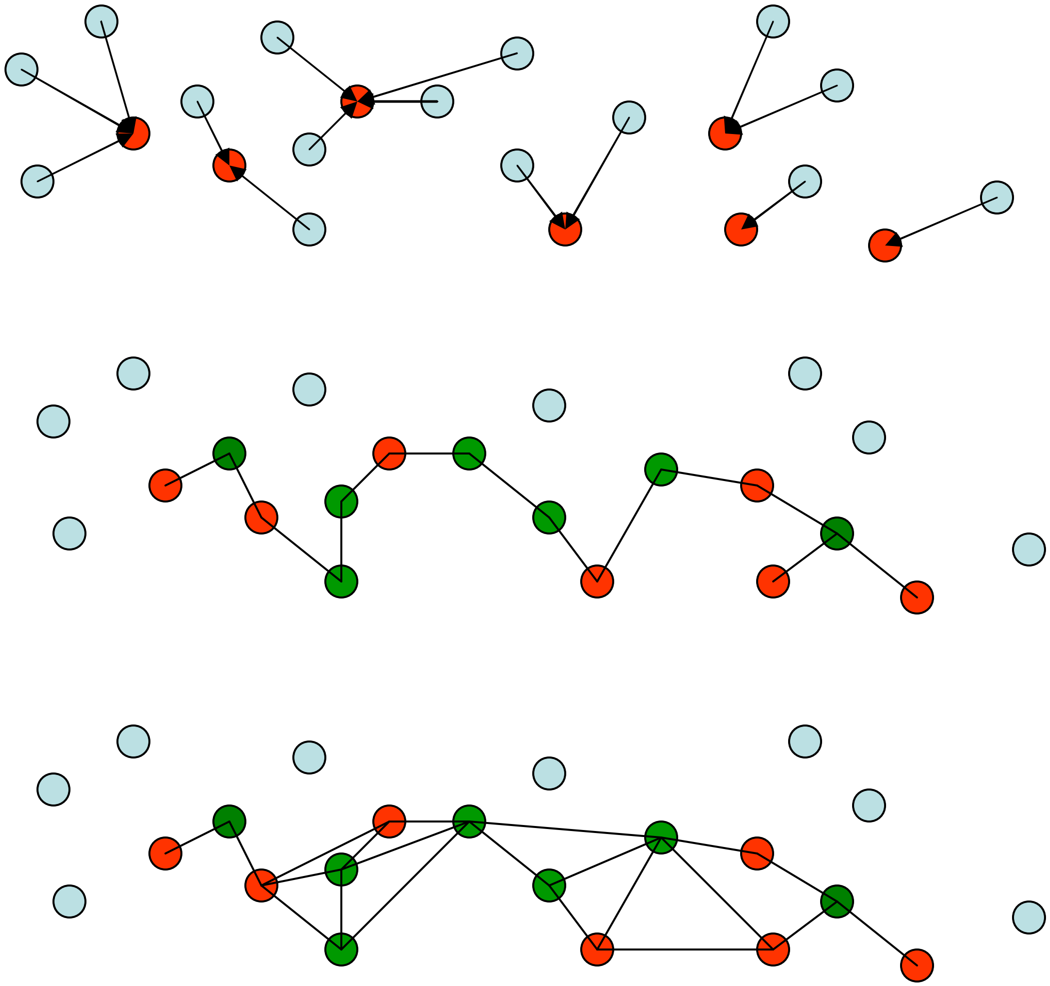
- For two nodes u, v that are k hops away, there is a path through clusterheads and gateways with at most 3k+2 hops.



Shortest path

- Construct RDG on clusterheads and gateways, which have constant bounded density.

A Routing Graph Sample



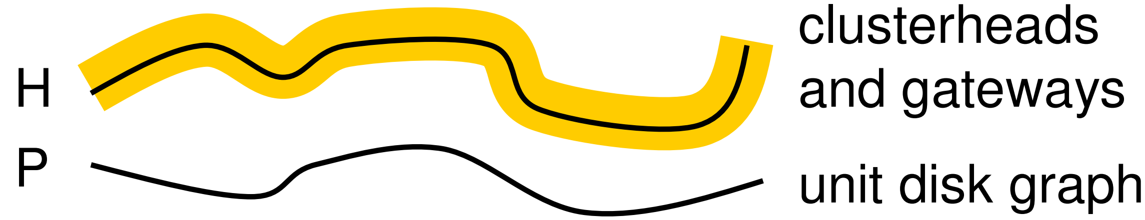
Select clusterheads

Clusterheads select gateways

RDG on clusterheads & gateways

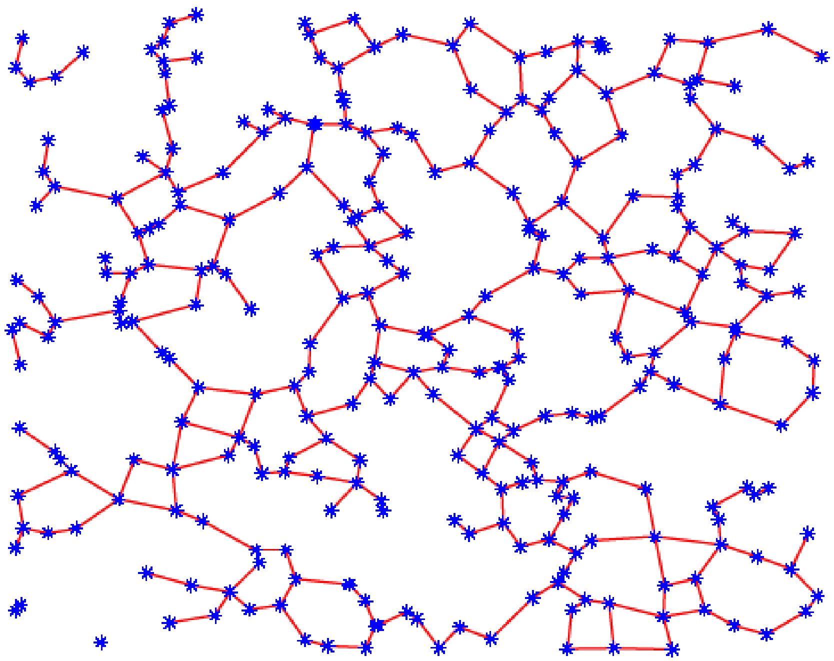
Restricted Delaunay graph

- Claim: (RDG on clusterheads and gateways + edges from clients to clusterheads) is a constant hop spanner of the original UDG.

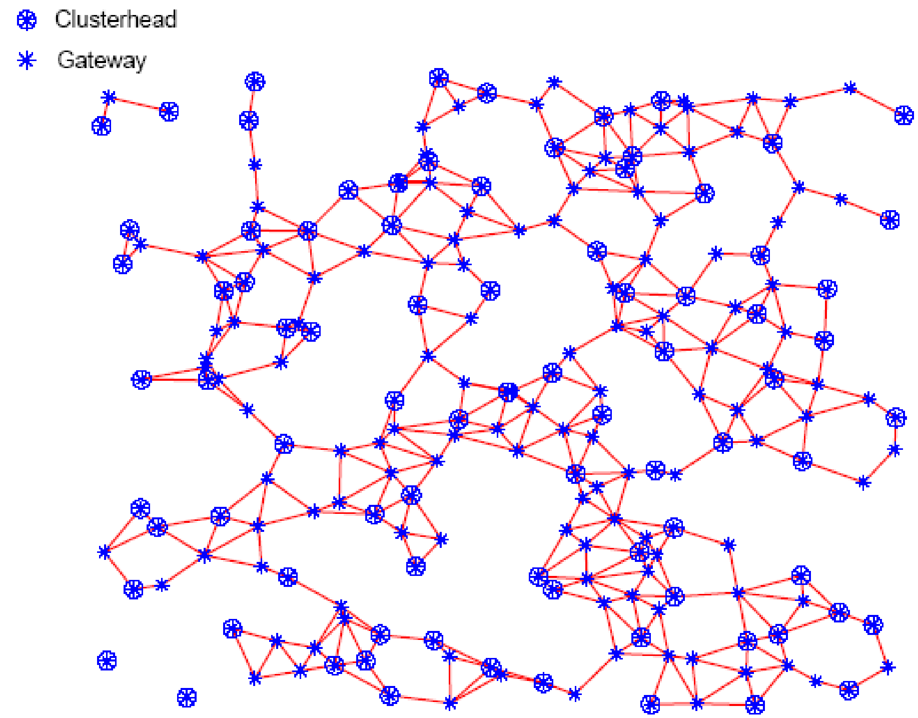


- Proof sketch:
  - The shortest path P in the unit disk graph has k hops.
  - Through clusterheads and gateways ∃ a path Q with ≤ 3k+2 hops.
  - Q's total Euclidean length is ≤ 3k+2.
  - The shortest path on the RDG, H, has Euclidean length ≤ 2.42×(3k+2).
  - By constant density property a region with width 1 and length 2.42×(3k+2) has O(k) nodes inside. So # hops of H is O(k).
  - This concludes the hop spanner property.

Restricted Delaunay graph

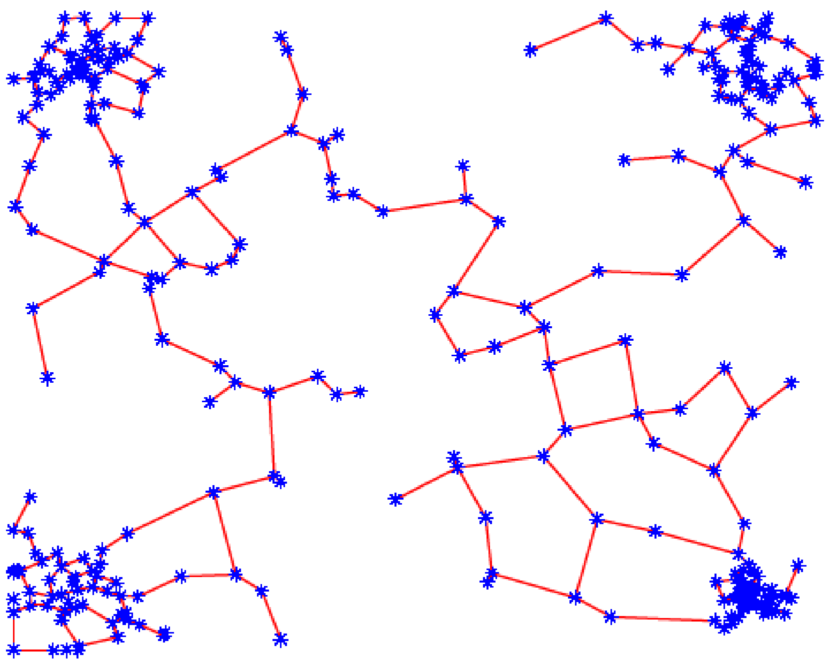


RNG

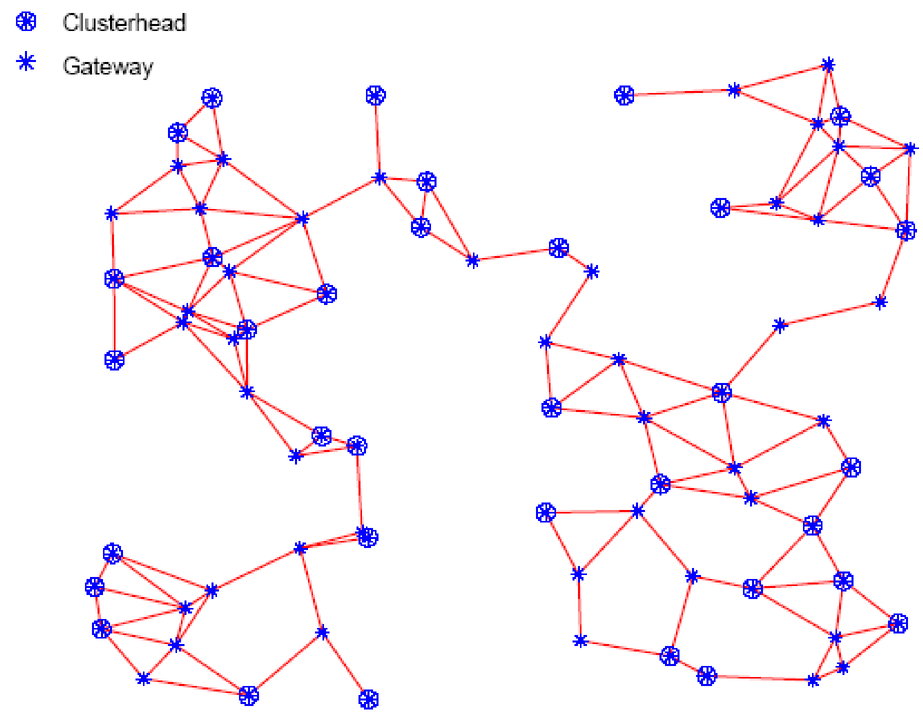


RDG

Restricted Delaunay graph



RNG



RDG

Tackle problem II:
Improve face routing to find a short path &
Geographic routing in practice

Papers

Geographic routing in practice:

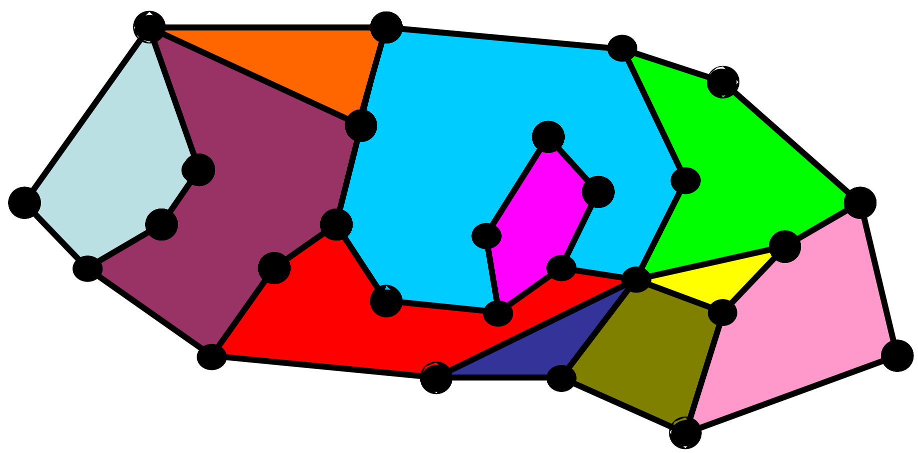
- Kim, Y.-J., Govindan, R., Karp, B., and Shenker, S., **On the Pitfalls of Geographic Face Routing**, DIAL-M-POMC'05.

Virtual coordinates:

- Ananth Rao, Christos Papadimitriou, Scott Shenker, and Ion Stoica, **Geographical routing without location information**, Proc. MobiCom'03, pages 96 - 108, 2003.

Overview of geographical routing

- Routing with geographical location information.
  - Greedy forwarding.
  - If stuck, do face routing on a planar sub-graph.



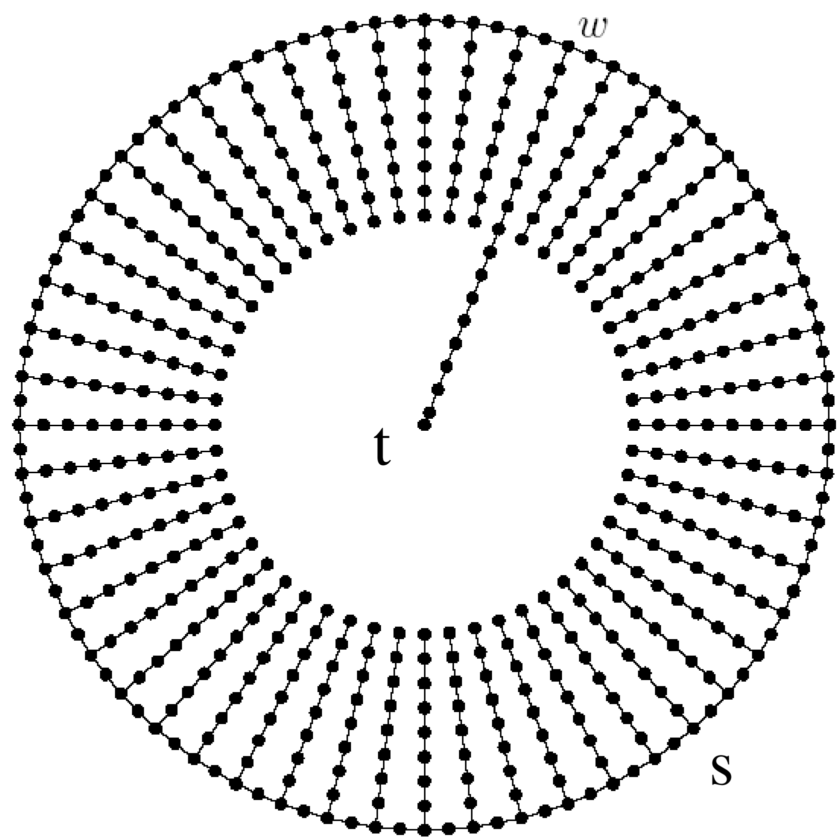
Overview of last lecture

- How to find a planar subgraph?
  - Use distributed construction: relative neighborhood graph, Gabriel graph, etc.
  - A planar subgraph that contains a short path: restricted Delaunay graph: short Delaunay edges.
- Big problem: how is the performance of geo-routing?
  - Can we always find a short path?

Bad news: Lower bound of localized routing

- Any deterministic or randomized localized routing algorithm takes a path of length $\Omega(k^2)$, if the optimal path has length k.

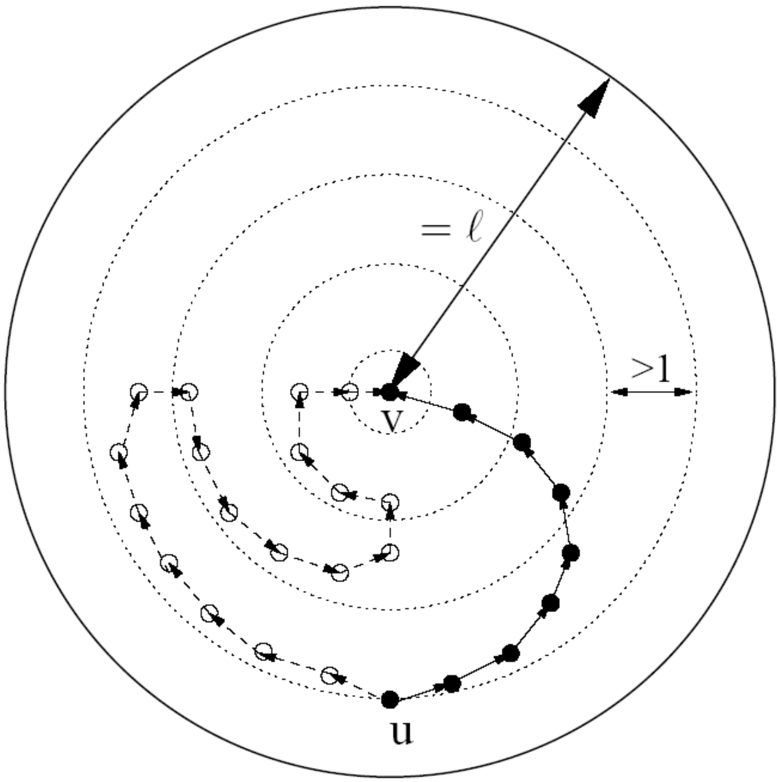
- The adversary decides where the chain wt is. Since we store no information on nodes, in the worst case we have to visit about $\Omega(k)$ chains and pay a cost of $\Omega(k^2)$.



Good news: greedy forwarding is optimal

- If greedy routing gets to the destination, then the path length is at most $O(k^2)$, if the optimal path has length k.

- |uv| is at most k. On the greedy path, every other node is not visible, so they are of distance at least 1 away. By a packing lemma, there are at most $O(k^2)$ nodes inside a disk of radius k.



How is face routing? How is greedy + face routing?