

An open problem on greetings

CSE 642 - Algorithms reading group

Stony Brook University

Scribes: Giordano Fusco, Mohammad T. Irfan ({fusco, mtirfan}@cs...)

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1 Introduction

An interesting open problem was introduced by Professor Michael Bender in the reading group. The problem arises in the setting of a social gathering and later on it was found that the problem might have applications in several areas of Computer Science. Let us first give a brief introduction to the social setting of the problem and then give references to a few application areas.

In a social meeting, especially in some European countries, whenever a person enters or leaves the meeting he/she has to greet others by kissing, hugging, or shaking hands. The way in which this greeting is conducted varies from one country to another. For example, in some countries it is usual to kiss twice on the cheeks and in some others thrice. In this social setting we can ask the following questions: what is the minimum time required to complete the greetings? Or, what is the maximum number of greetings that can happen in a specified period?

Interestingly, these questions also arise in some areas of Computer Science. In a sensor network, two sensors has to exchange information, which is quite similar to two persons greeting each other. Can we give a strategy so that sensors in a network can complete a phase of information exchange in the minimum time?

Generic Problem. We are given n persons located at n points on the plane, and each of them is allowed to move. Moving has an associated cost $T_m(i, j)$, which represents the time to go from point i to point j (we assume that they move at the same speed). Each person is required to meet all others to greet them. Each greeting has a cost T_g .

We can ask some questions, such as:

- What is the minimum time to complete all greetings?
- What is the maximum number of greetings that can be done in a fixed time?

- What is the minimum total distance to complete all greetings?
- What is the minimum time for each person to greet everyone else?

1.1 Alternative formulations

We can give several alternative formulations of this problem.

Continuous vs Discrete. In the continuous version, each person is located at a point in 2D space, and can move in any direction. In the discrete version, each person is located in a cell of a $M \times N$ chessboard-like configuration, where $M, N > 0$. In this case, a movement is subject to the following restrictions:

- each cell can contain at most one person;
- each person can move to an adjacent free cell;
- two adjacent persons can swap positions;
- a contiguous sequence of m persons can shift by one cell simultaneously;
- the greeting happens only between neighbors.

Time measurement. Time can be measured by considering only the movement time, or only the greetings time, or both of them.

Multiple greetings vs Paired greetings. In the multiple greetings model, two or more persons are allowed to greet at the same time if they are “nearby” each other. In the paired greetings model, greeting happens only between two persons at a time.

Offline vs Online. In the offline variation, there is a central agent that determines the sequence of moves to satisfy the objective before the start of any action. In the online version, all the persons act independently following a strategy.

Parallel vs Sequential. In the parallel case, there can be more than one move, or more than one greeting at the same time. In the sequential one, only one move or greeting is allowed at one time.

2 Approaches

Here we present some approaches that were discussed in the reading group.

General hug. In the continuous case, if we allow multiple greetings at the same time, one approach is to gather at some point and greet all at the same time, like a “general hug”.

Collection and milling. It was suggested that the problem can have two phases. One is collection and the other is milling. In the collection phase, the persons will form a certain configuration, and in the milling phase they will greet each other.

Double line. Suppose that everyone is arranged on two parallel lines. Then everyone greets the one opposite to her/him. After that, each person shifts by one position cyclically (the person at the end of a line goes at the beginning of the other line). This approach might require to include an empty cell in order to work in the discrete case. In the parallel model, the total time of moving and greetings is linear.

Divide and conquer. Can there be a hierarchical organization of movements and greetings? This approach might be useful if the original configuration contains some clusters of people located far apart from the others.

3 Complexity of the problem

Can this problem be solved in polynomial time?

Here we give the sketch of a polynomial time algorithm for the discrete case. We can think about a similar solution for the continuous one.

Let T_s be the time to move by one cell, and let T_g the time for one greeting. Let R be the radius of the smallest circle containing all initial positions. Each person at a time moves to the center of the circle (this requires up to R steps). Note that it is always possible to reach the center because two adjacent persons are allowed to swap each other. In turn, everyone else goes towards the center to greet the person currently there. This requires up to $(n - 1)R$ steps and $(n - 1)$ greetings (we may want to avoid duplicate greetings, but here we are just trying to give an upper bound on the time). Then, an upper bound on the time is $n(RT_s + (n - 1)(RT_s + T_g))$ which is $O(n^2(RT_s + T_g))$.

This shows that the problem can be solved in polynomial time. However the algorithm above makes use of the fact that two persons can swap their positions. Now suppose that they are not allowed to. If the grid is wide enough there should not be any problem. What happens if there is only one free cell in the grid, like the “15 puzzle”?

4 Similar problems

A few similar problems were also discussed in the reading group. A paper of Eric Demaine *et. al.* on minimum motion was referenced. Another problem on lighting torches was discussed. In the torch-lighting problem there are two sets of persons: one with the torches lit and one with the torches off. When a person with the torch off meets one with the light, his torch is lit. The goal is to light on all the torches with some specific objective.