Discovery of Sensor Network Geometry

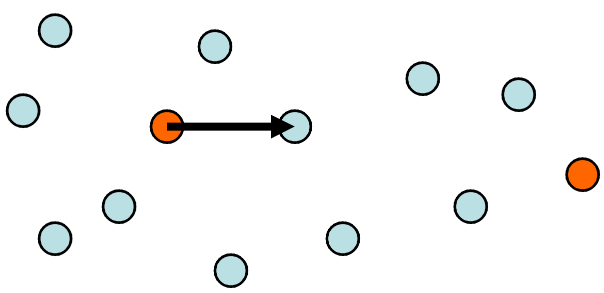
Jie Gao

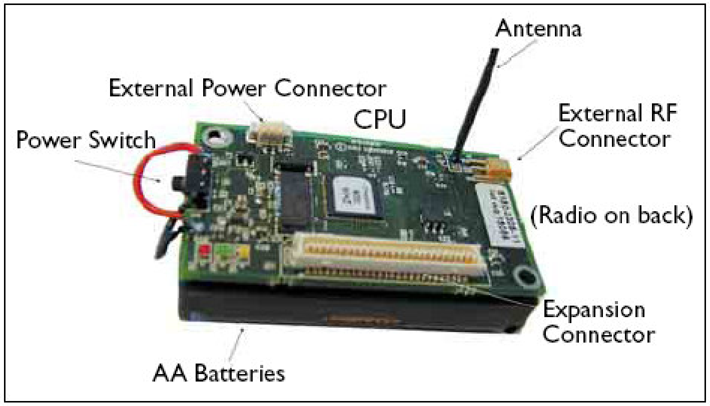
State University of New York at Stony Brook

Joint work with Sol Lederer and Yue Wang

Localization of sensor nodes

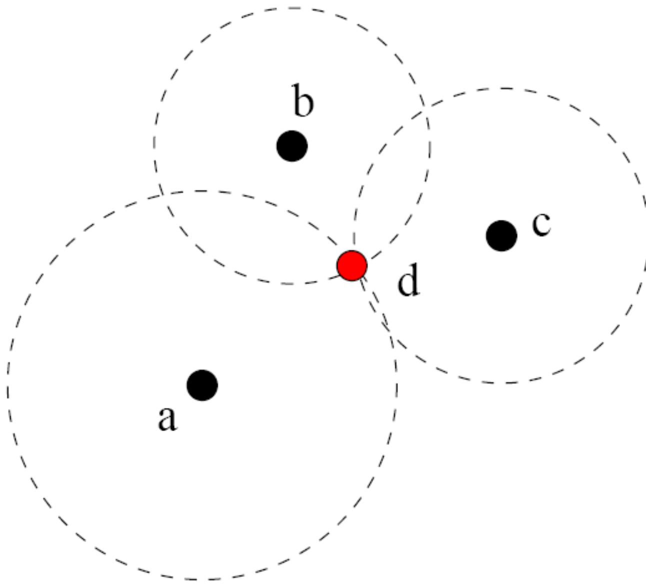
- Find out where the sensors are.
  - For the integrity of sensor data.
  - Useful for network organization and functioning (e.g., geographical routing).





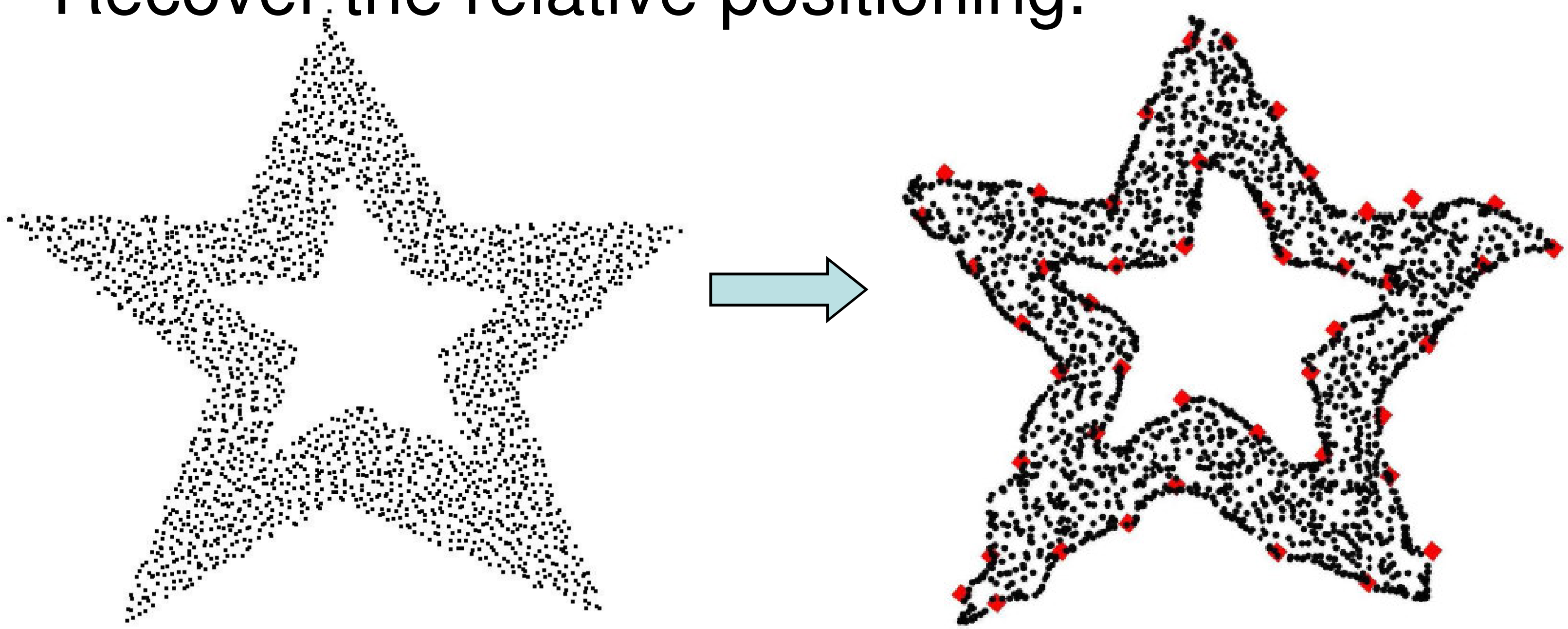
Existing localization techniques

- GPS
  - expensive, large form factor, does not work in-door.
- Anchor-based localization.
  - Anchors have GPS.
  - Non-anchor nodes measure distances to anchors.
- This talk:
  - Anchor-free localization with connectivity information only.



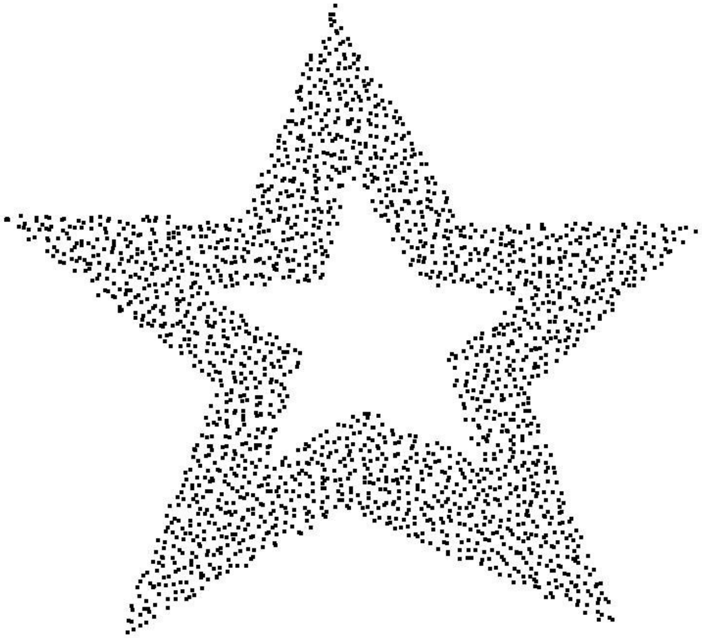
Localization of large-scale sensor network

- Large-size sensor field with complex geometry.
- Nearby nodes are able to communicate.
- Use connectivity information only.
- Recover the relative positioning.

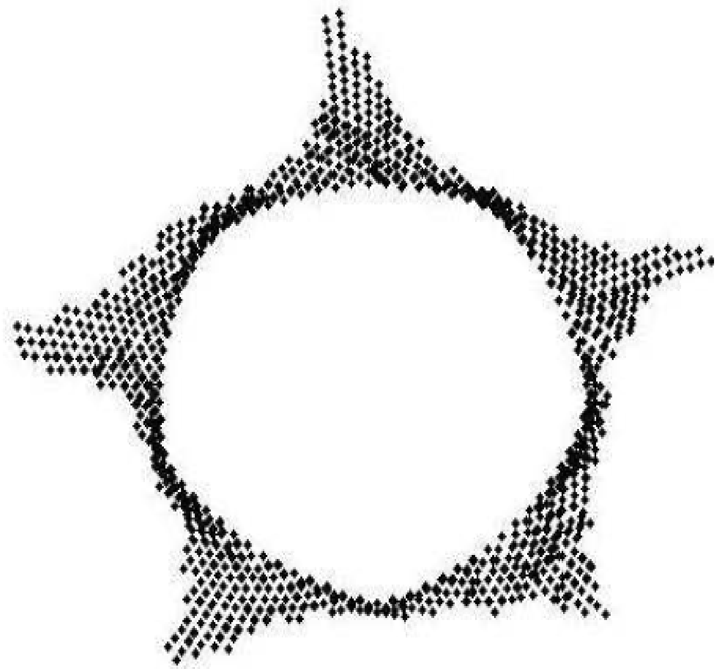


Comparison: multi-dimensional scaling

Ground Truth

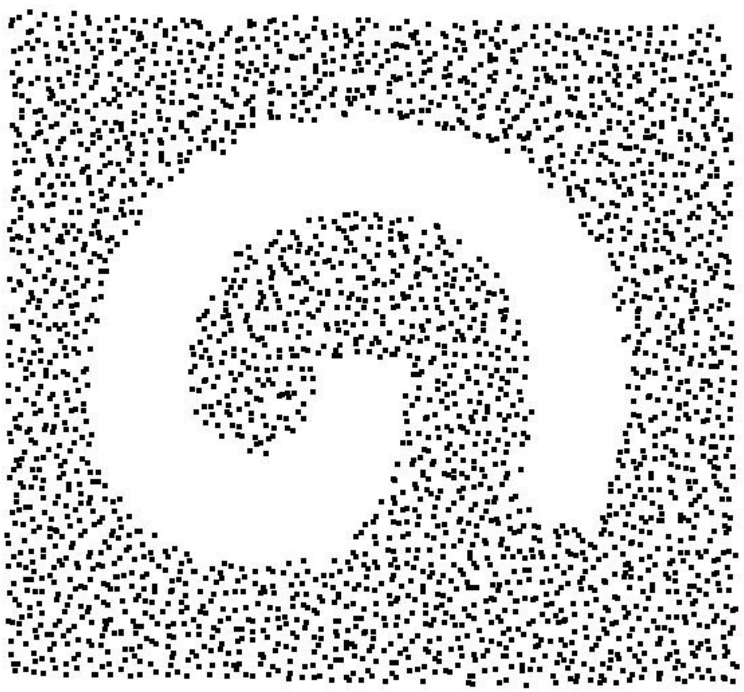


Multi-dimensional Scaling

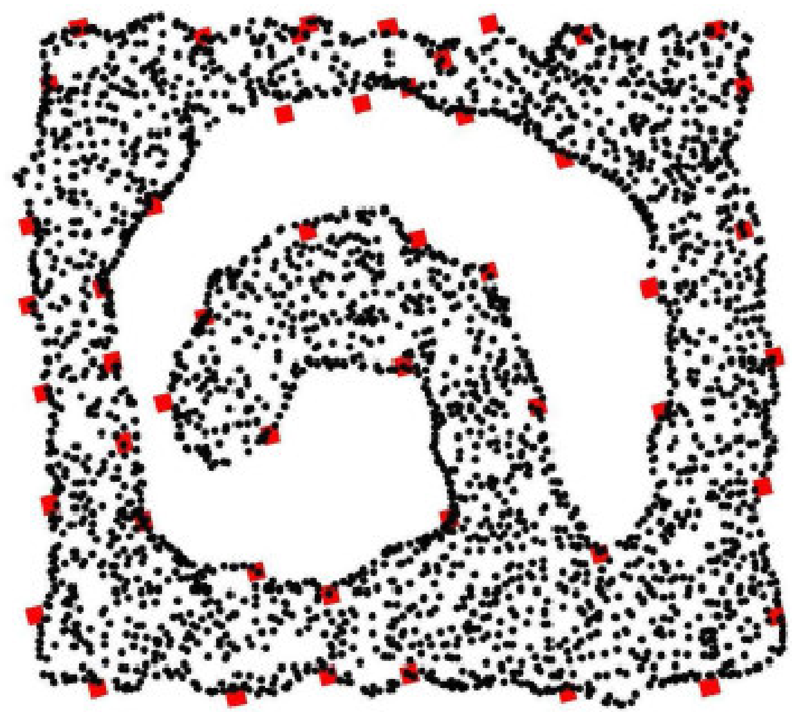


Our results

Ground Truth

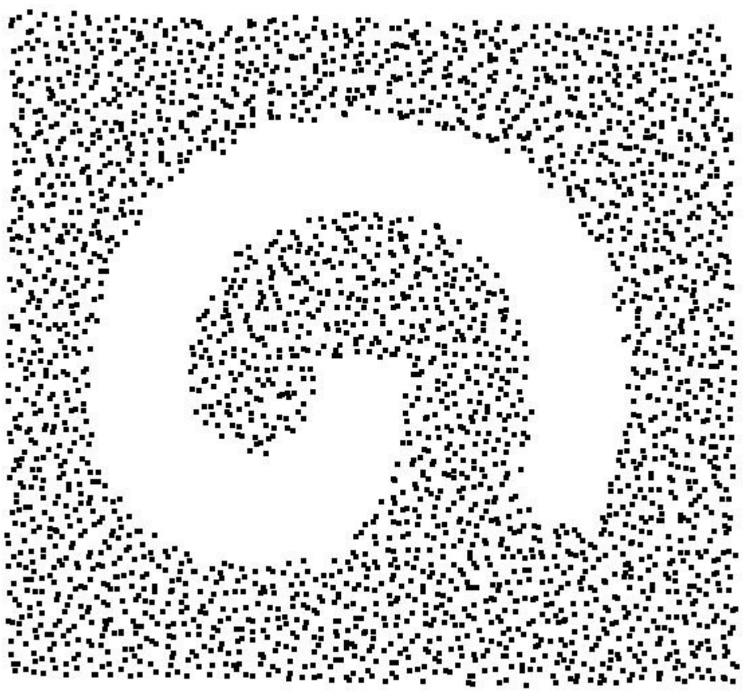


Our localization

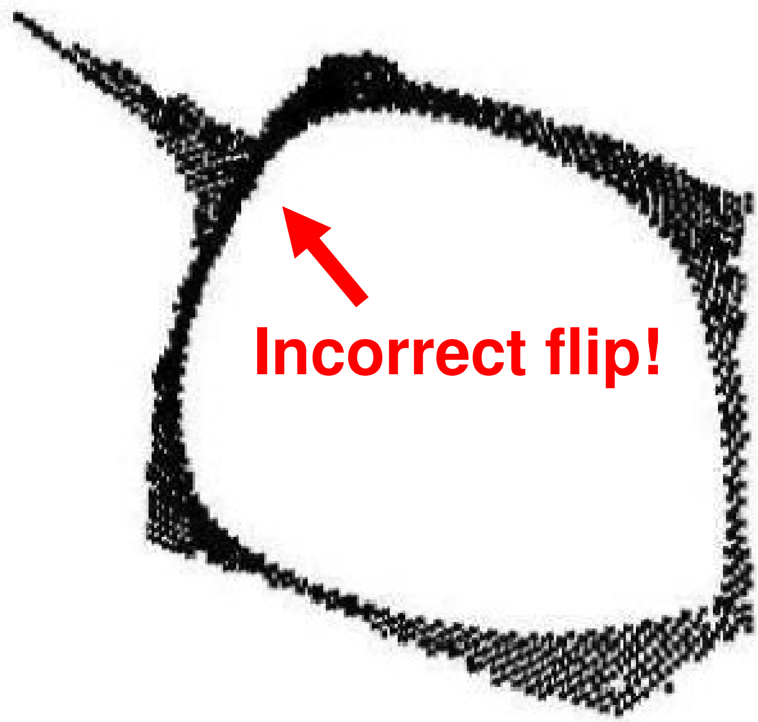


Comparison: multi-dimensional scaling

Ground Truth

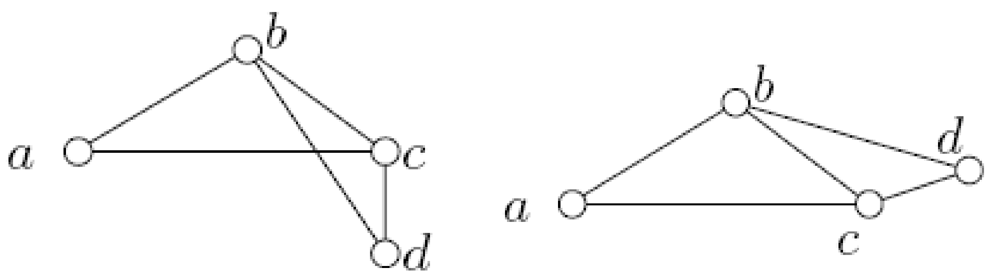


MDS

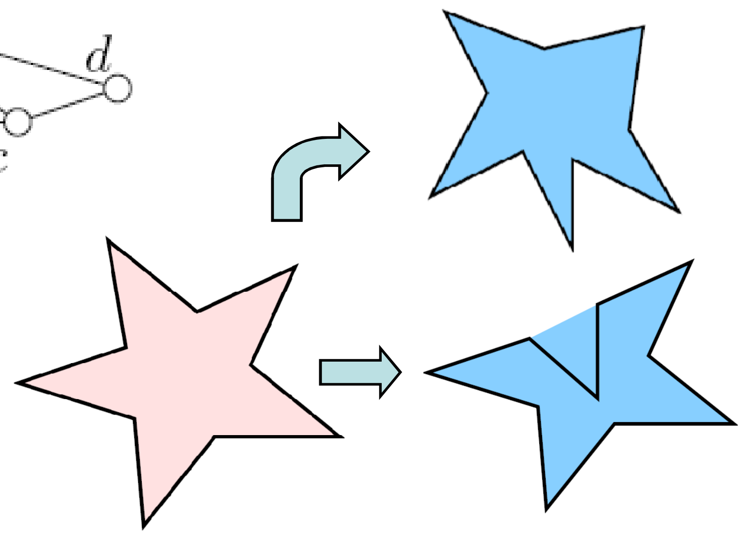


The challenge: flip ambiguity

- Given two triangles with fixed edge length, one can flip one triangle relative to the other.

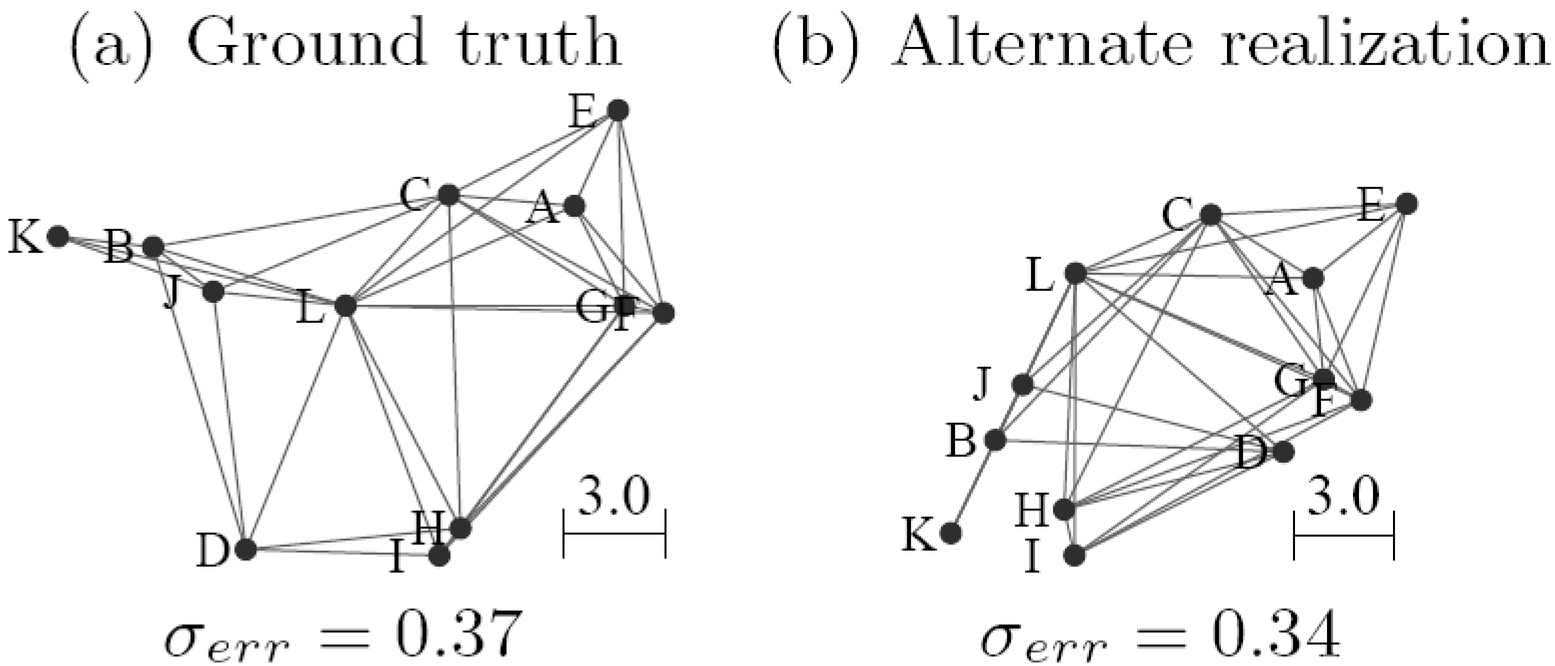


- Incorrect global flip.



In practice…

- Many optimization-based localization algorithms get stuck in local minimum.



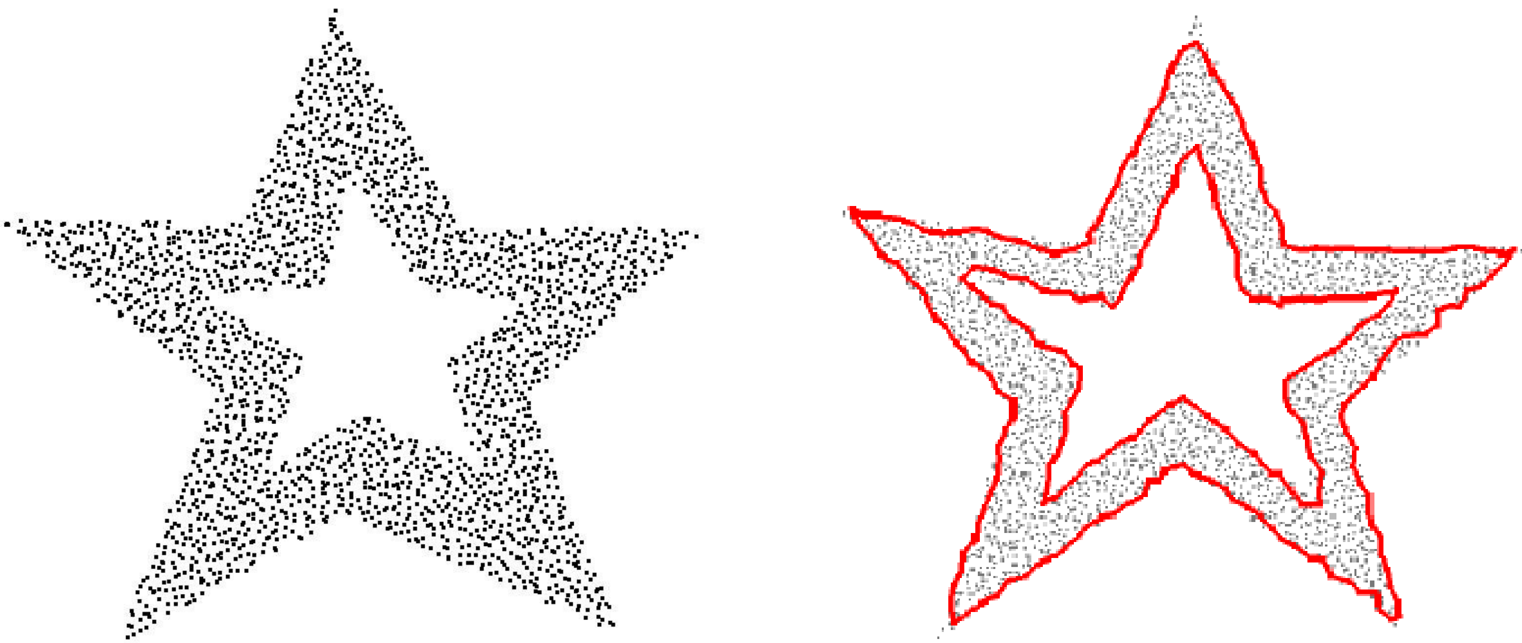
Reference: Anchor-Free Distributed Localization in Sensor Networks, Nissanka B. Priyantha, Hari Balakrishnan, Erik Demaine, and Seth Teller,Technical Report MIT-LCS-TR-892, 2003.

Outline

- The localization algorithm.
- Why we do not have flip ambiguity.
- Theoretical foundations.

Our algorithm

1. Detect network boundary.



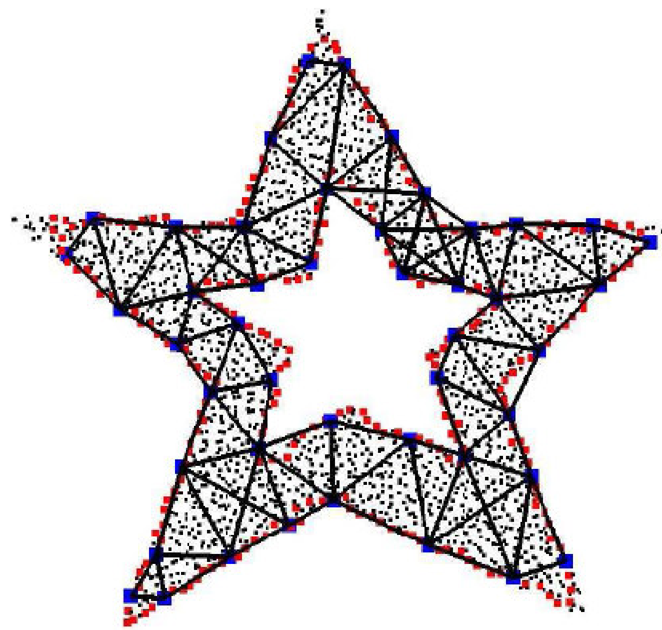
Existing algorithms by Sandor Fekete & Alexandar Kroller, Stefan Funke, our group.

Our algorithm, cont.

2. Select landmarks on the boundary.

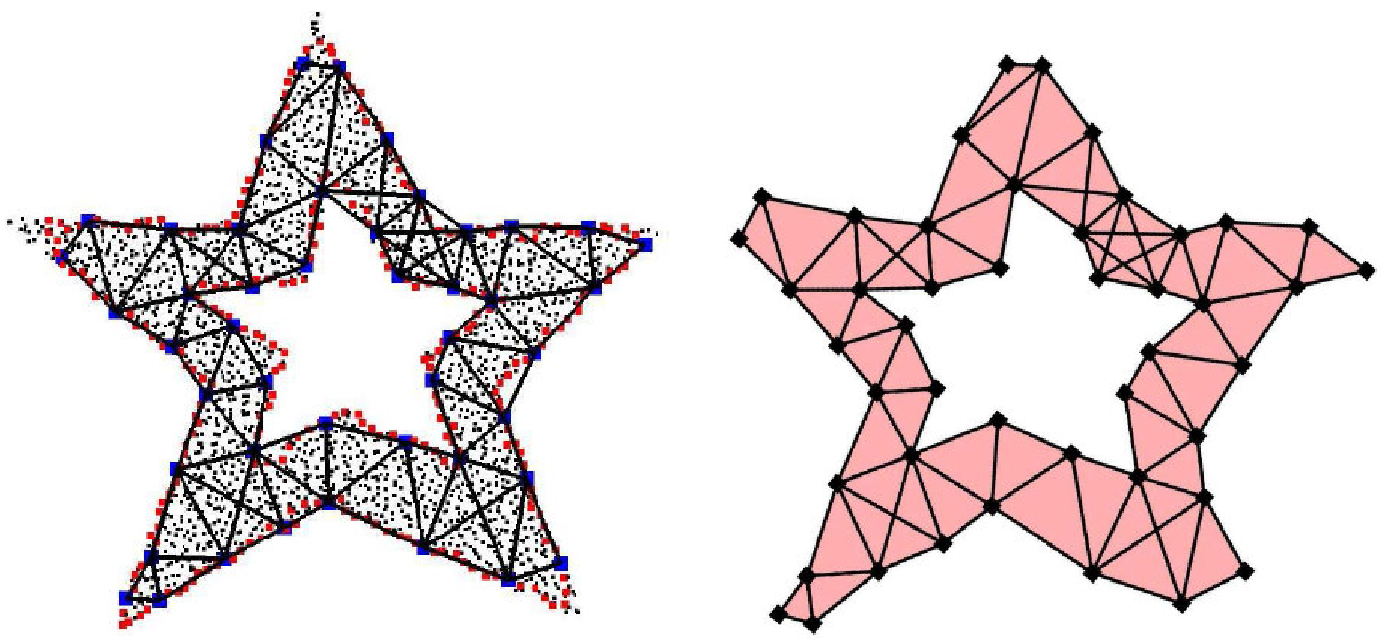
Landmark Voronoi diagram:
Nodes identify closest landmark

Combinatorial Delaunay complex
Neighboring landmarks connected by an edge



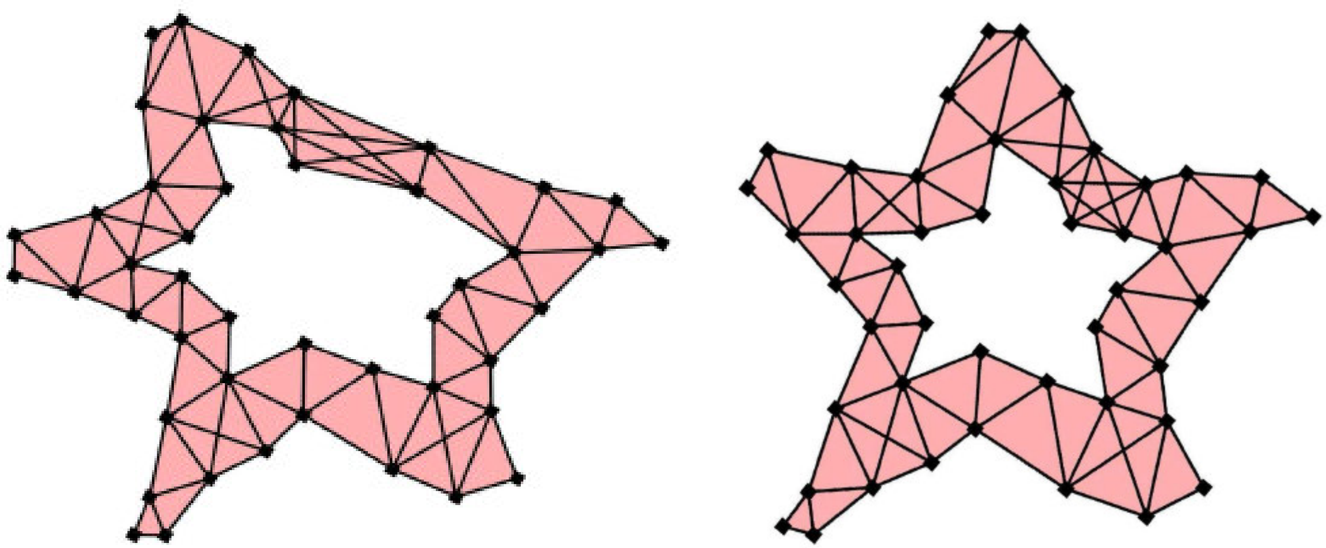
Our algorithm, cont

3. Embed the combinatorial Delaunay complex (purely combinatorial, no geometry), find a realization (simplicial complex, with geometry)



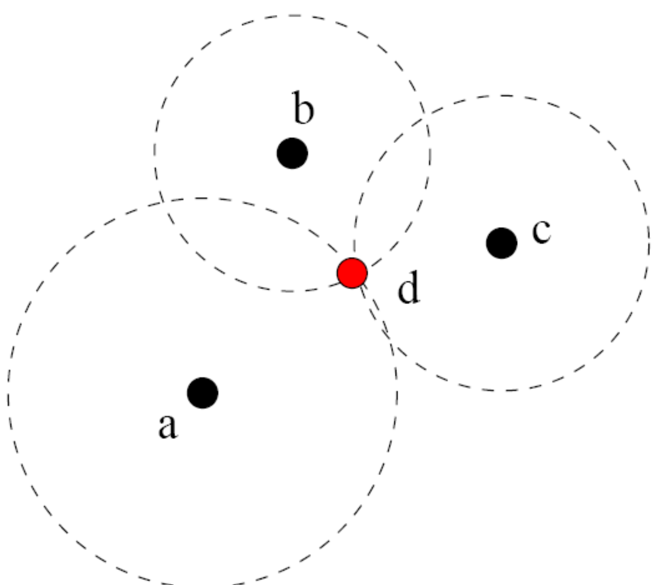
Our algorithm, cont

3. Embed the landmarks.
   1. Each edge has a length = min hop count.
   2. Glue the triangles incrementally.
   3. Do a mass-spring relaxation to smooth out errors.



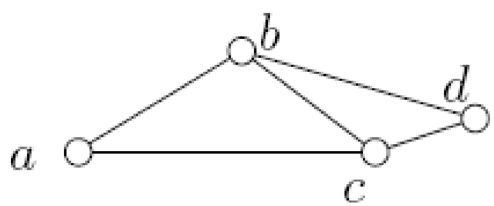
Our algorithm, cont

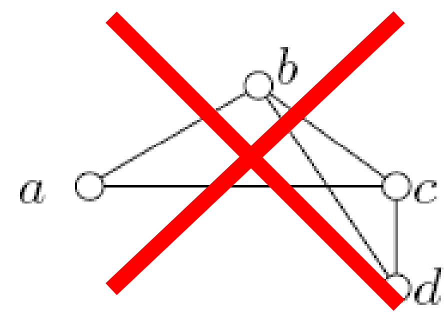
4. Embed the rest of the nodes.
   - Each node embeds itself with hop-count distances to nearby 3 landmarks.



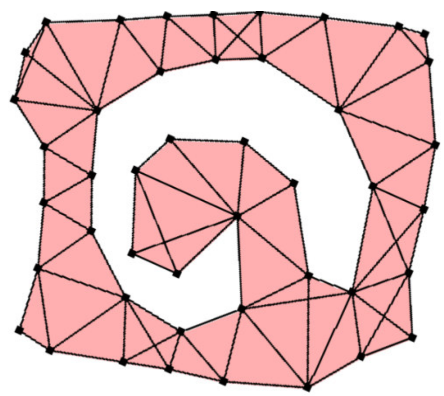
Now: no flip ambiguity of Delaunay triangles

- Two adjacent Delaunay triangles must be embedded as disjoint.
- Key insight: they are “solid”.





Ground truth



Our result

MDS

Abstract simplicial complex & simplicial complex

- Abstract simplicial complex (defined in terms of sets).
  - An edge: a set of two vertices.
  - A triangle: a set of three vertices, and so on.
  - An abstract simplicial complex: a collection A of sets s.t. if α 2 A and β μ α, then β 2 A.

Abstract simplicial complex & simplicial complex

- Abstract simplicial complex (defined in terms of sets).
- Simplicial complex (with geometry): any two simplices are either disjoint or intersect at a common face (subsimplex).

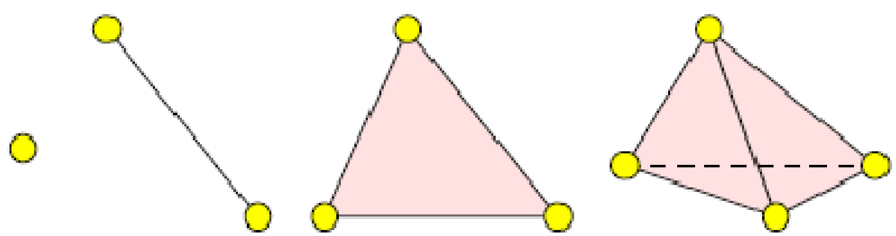
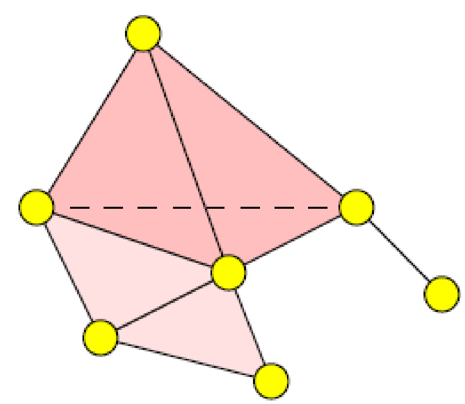
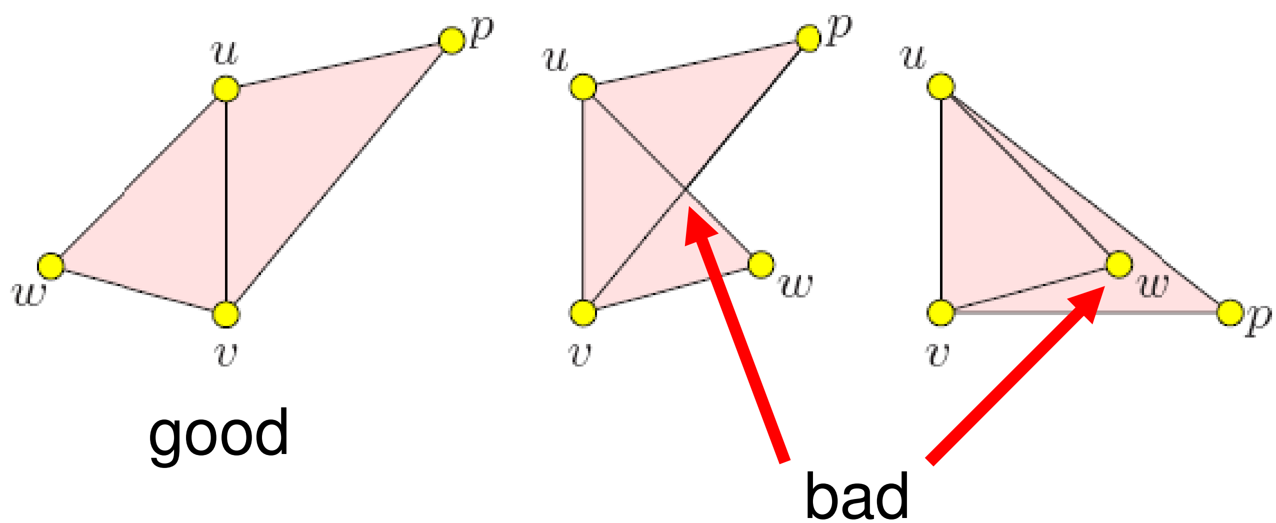


Figure 7. 0, 1, 2, 3-simplex in $\mathbb{R}^3$.



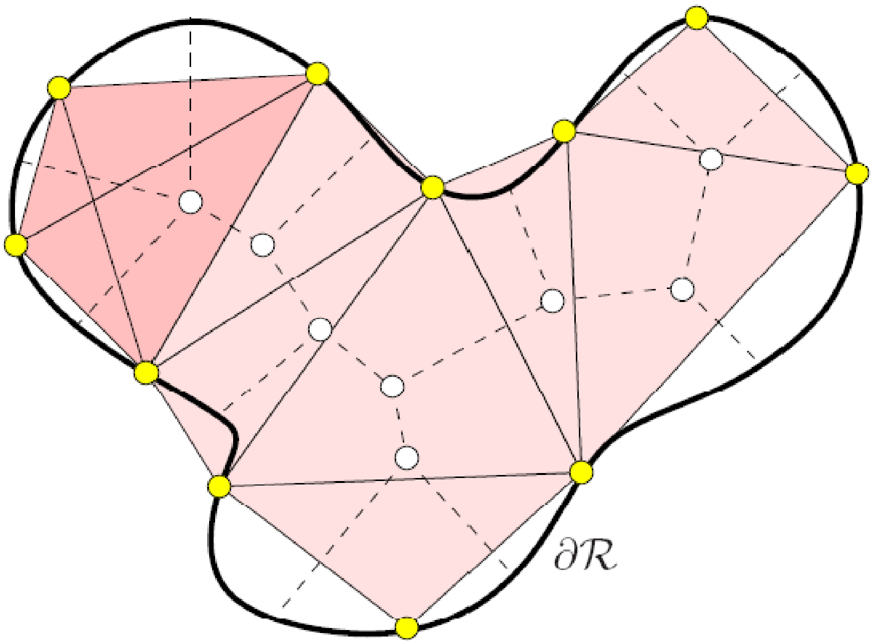
No flip ambiguity for Delaunay triangles

- Recall: we want to embed as a simplicial complex.
  - Intersection of two simplices is empty or a common face.

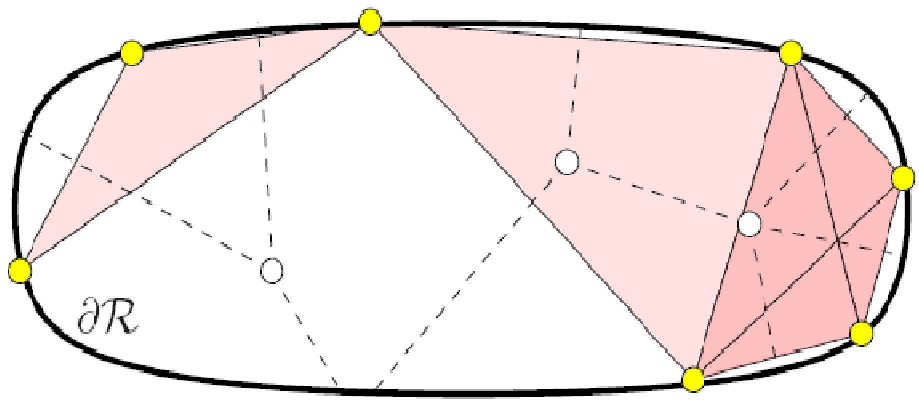


What we need: make sure the Delaunay complex is rigid.

- A graph is rigid if one can not deform the shape without changing the edge lengths.



Rigid

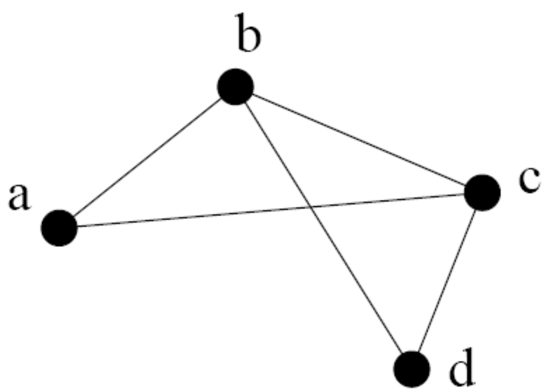


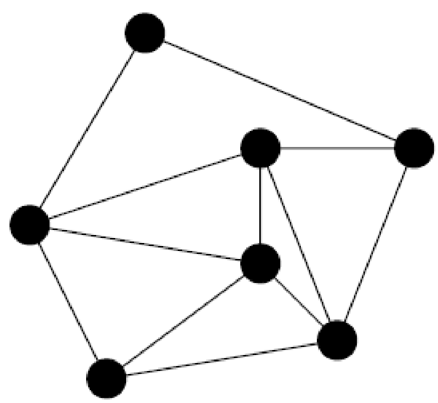
Non-rigid

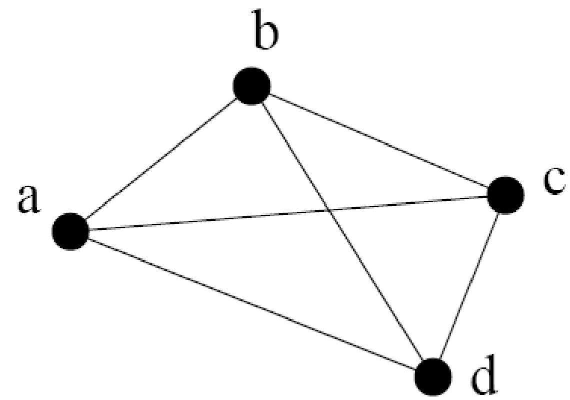
Need to select sufficiently dense landmarks!

2D rigidity: Laman condition

- A graph is rigid in 2D if and only if it contains a Laman graph on its vertices.
- A Laman graph has n vertices, 2n-3 edges and any subset of k vertices spans at most 2k-3 edges.

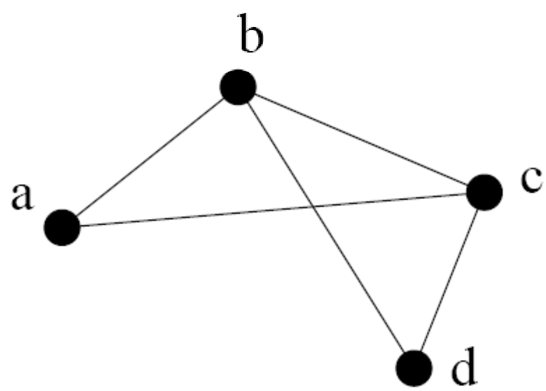


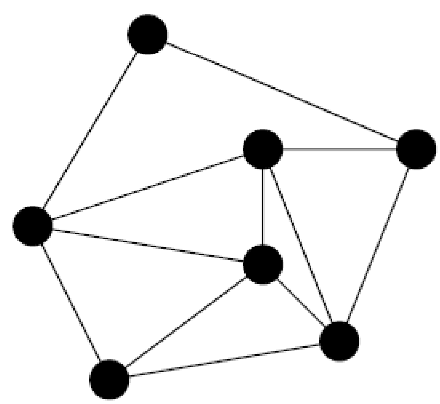




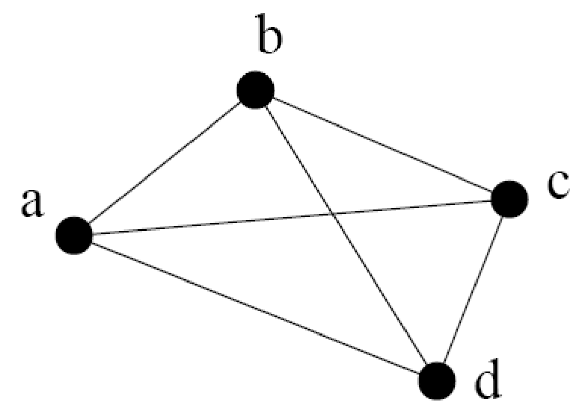
Rigidity and global rigidity

- A graph is globally rigid if it has a unique embedding in the plane.





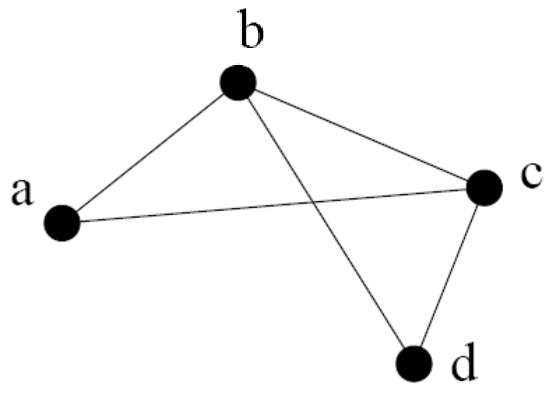
Rigid, not globally rigid

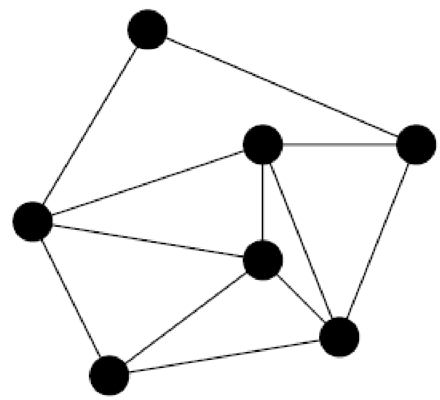


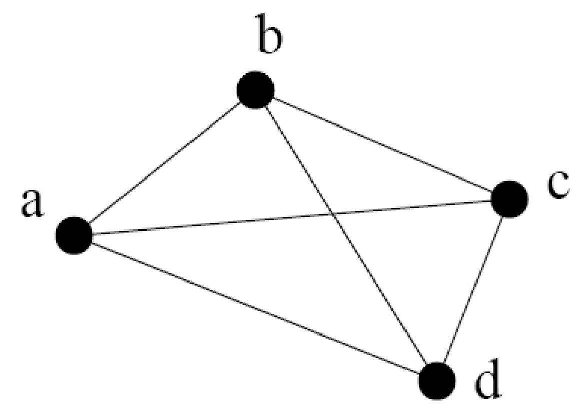
Globally rigid

Rigidity and global rigidity

- A graph is globally rigid in 2D iff it is 3-connected and redundantly rigid (rigid upon the removal of an edge).







Rigid, not globally rigid

Globally rigid

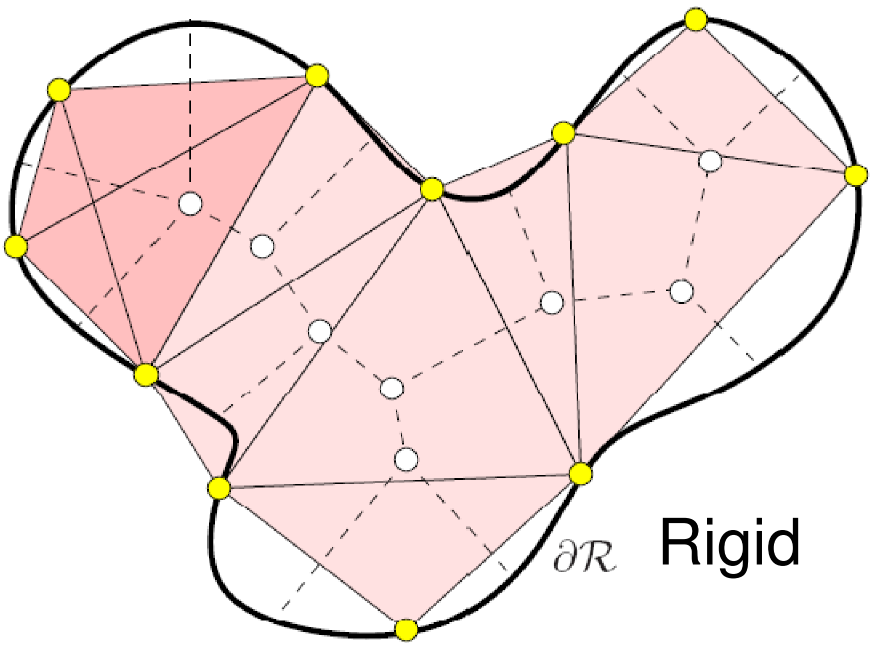
Remark on global rigidity

- To embed graphs, global rigidity is desired.
- To embed Delaunay complex, rigidity is enough.

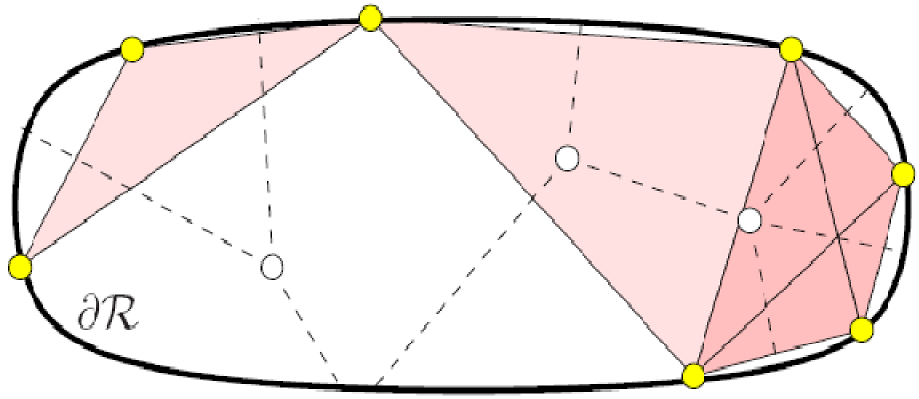
- Next: show the Delaunay graph is rigid given sufficiently dense landmarks.

Connect to the rigidity result

- If the Voronoi diagram (Voronoi edges and vertices) is connected in R, then the Delaunay graph is rigid.



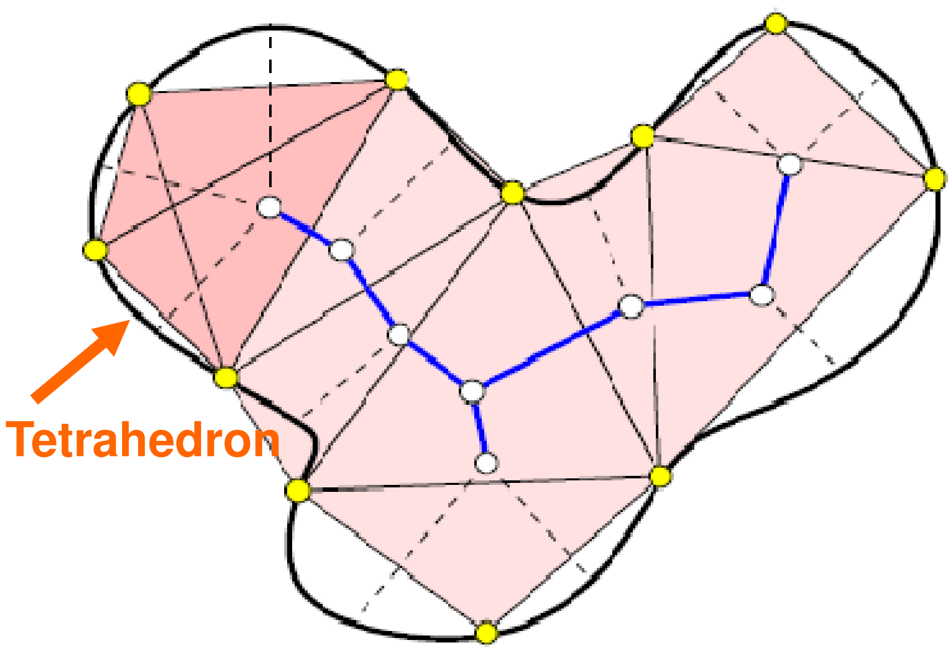
Rigid



Non-rigid

Proof by picture

- Show that the Delaunay graph contains a Laman graph.



Take a tree connecting Voronoi vertices.

Count:

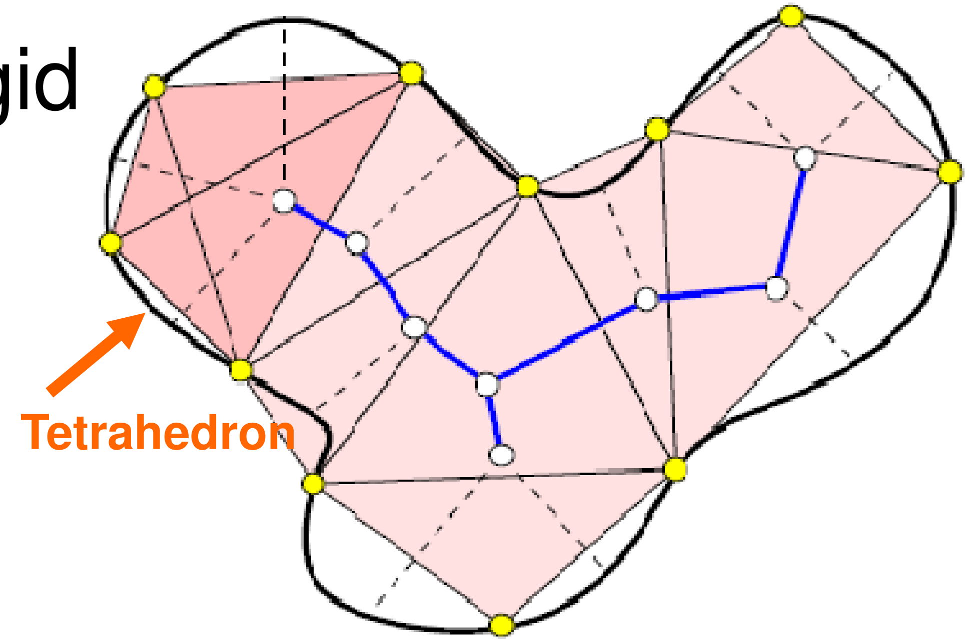
1. start with one Voronoi vertex (3 landmarks, 3 edges).
2. Add an adjacent Voronoi vertex (1 landmark, 2 edges).

Total: 2n-3 edges.

Also works for any subset of k vertices.

A remark on degeneracy

- For high-order simplices (k-simplex, k¸3), we get a complete graph on 4 or more vertices.
- Thus it is globally rigid



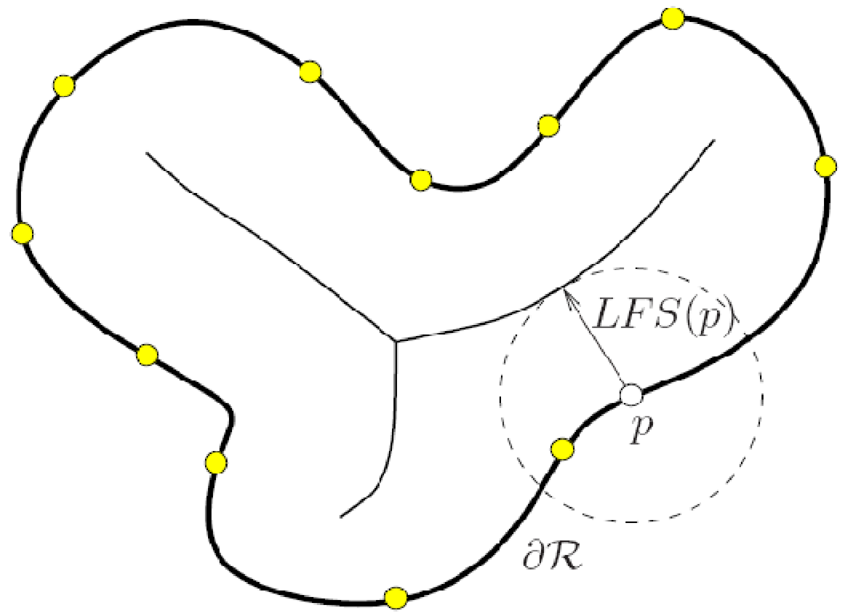
What is left

- When the landmarks are sufficiently dense (subject to local geometric feature), the Voronoi graph is connected.

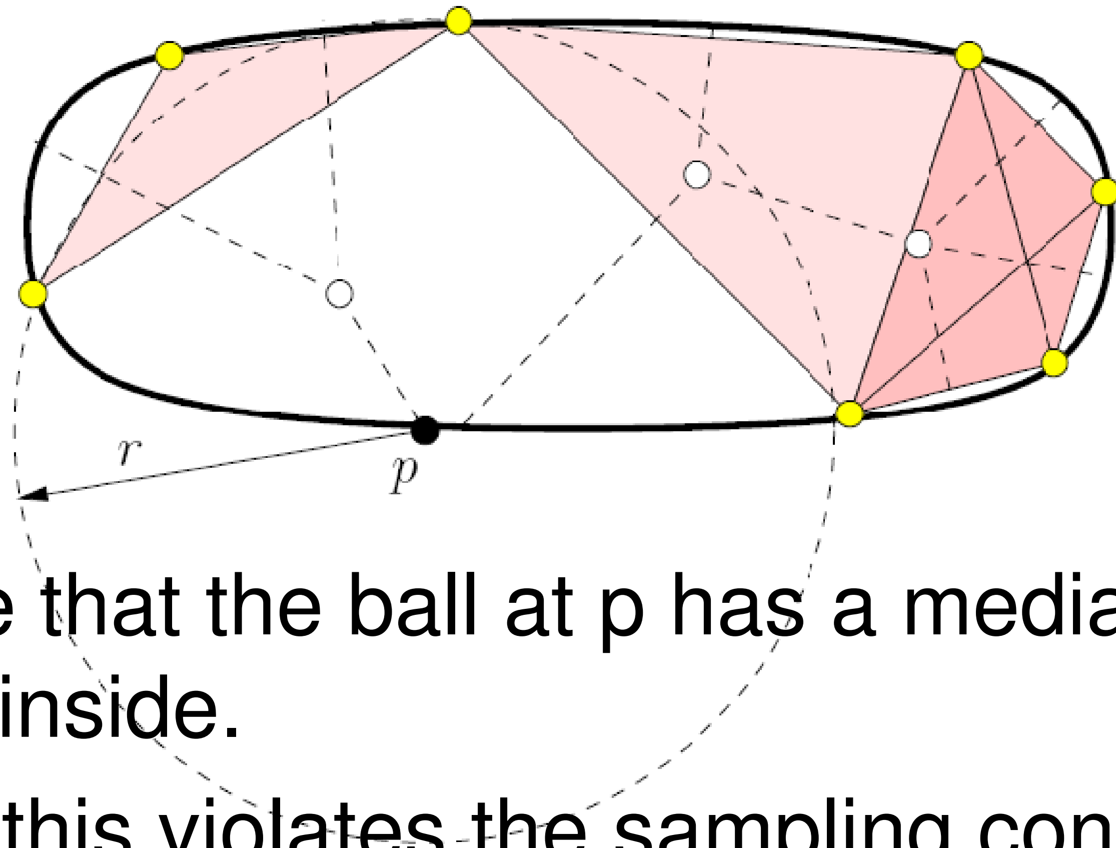
Medial axis, local feature size

- Medial axis: collection of points with two ore more closest points on the boundary.
- Local feature size: the smallest distance to the medial axis.
- Sampling criterion:

Each boundary point p has a landmark within LFS(p).

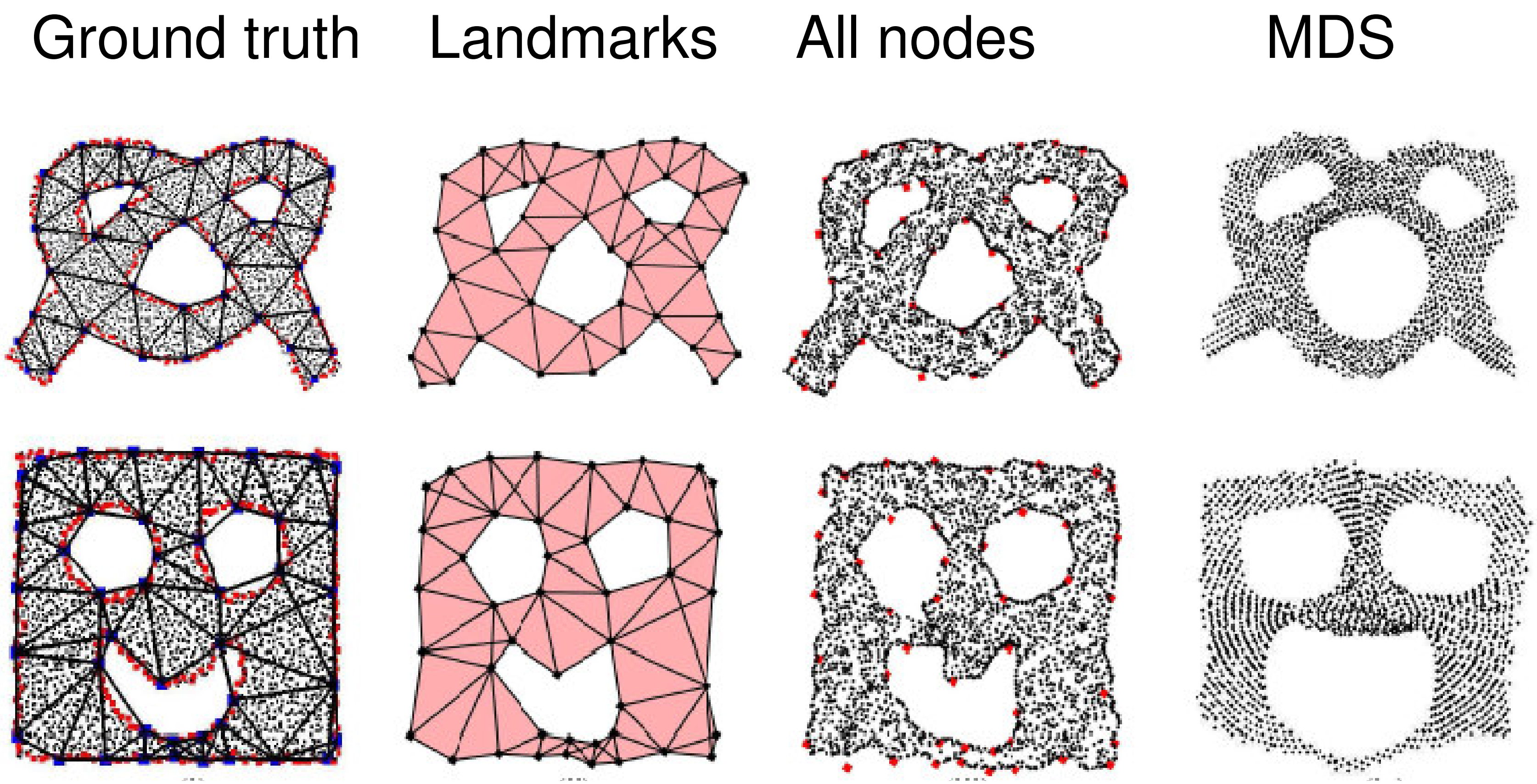


Proof by picture



- Argue that the ball at p has a medial axis point inside.
- Thus this violates the sampling condition because r>LFS(p).

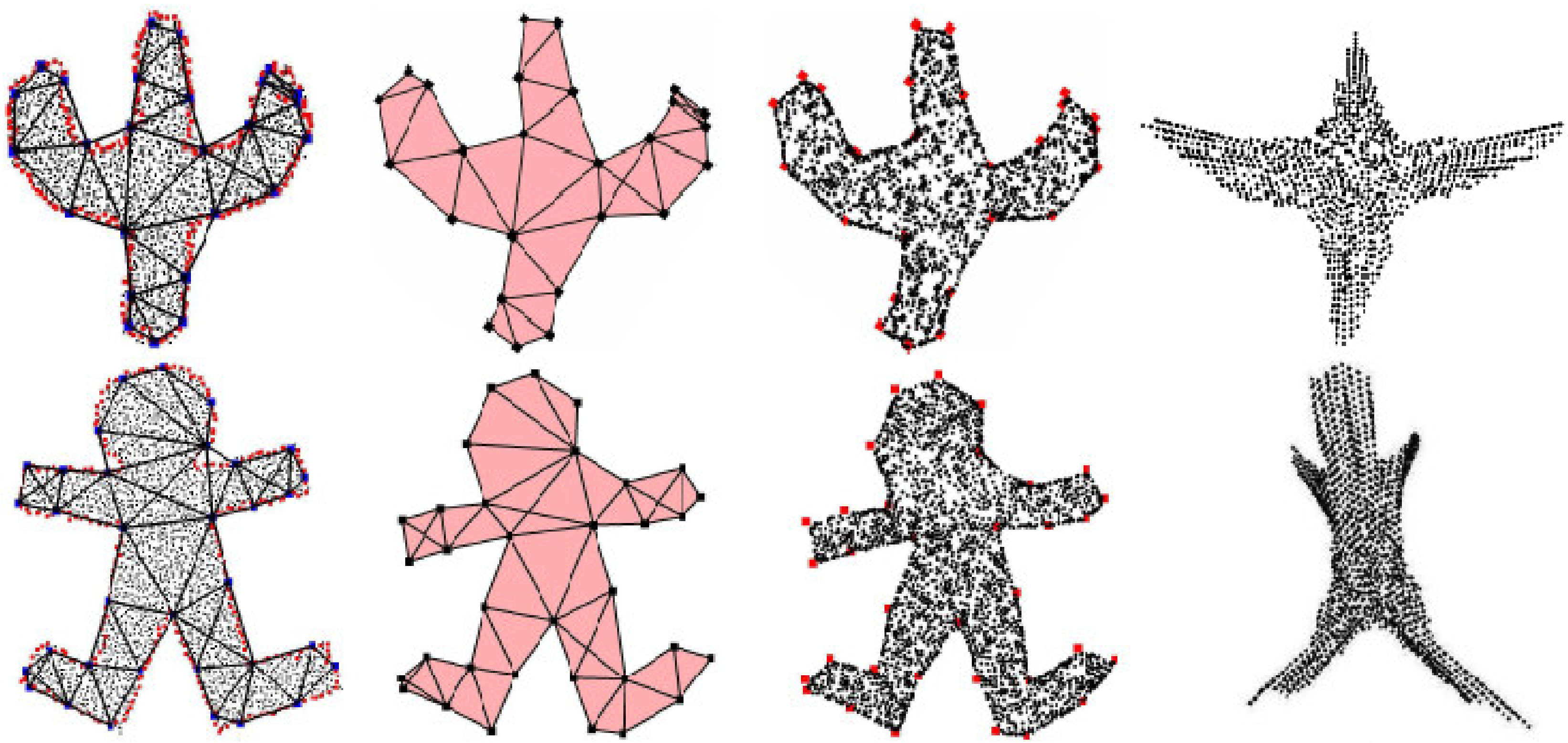
More results



Unit disk graph model, Node average degree 6~10.

More results

Ground truth Landmarks All nodes MDS



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More results

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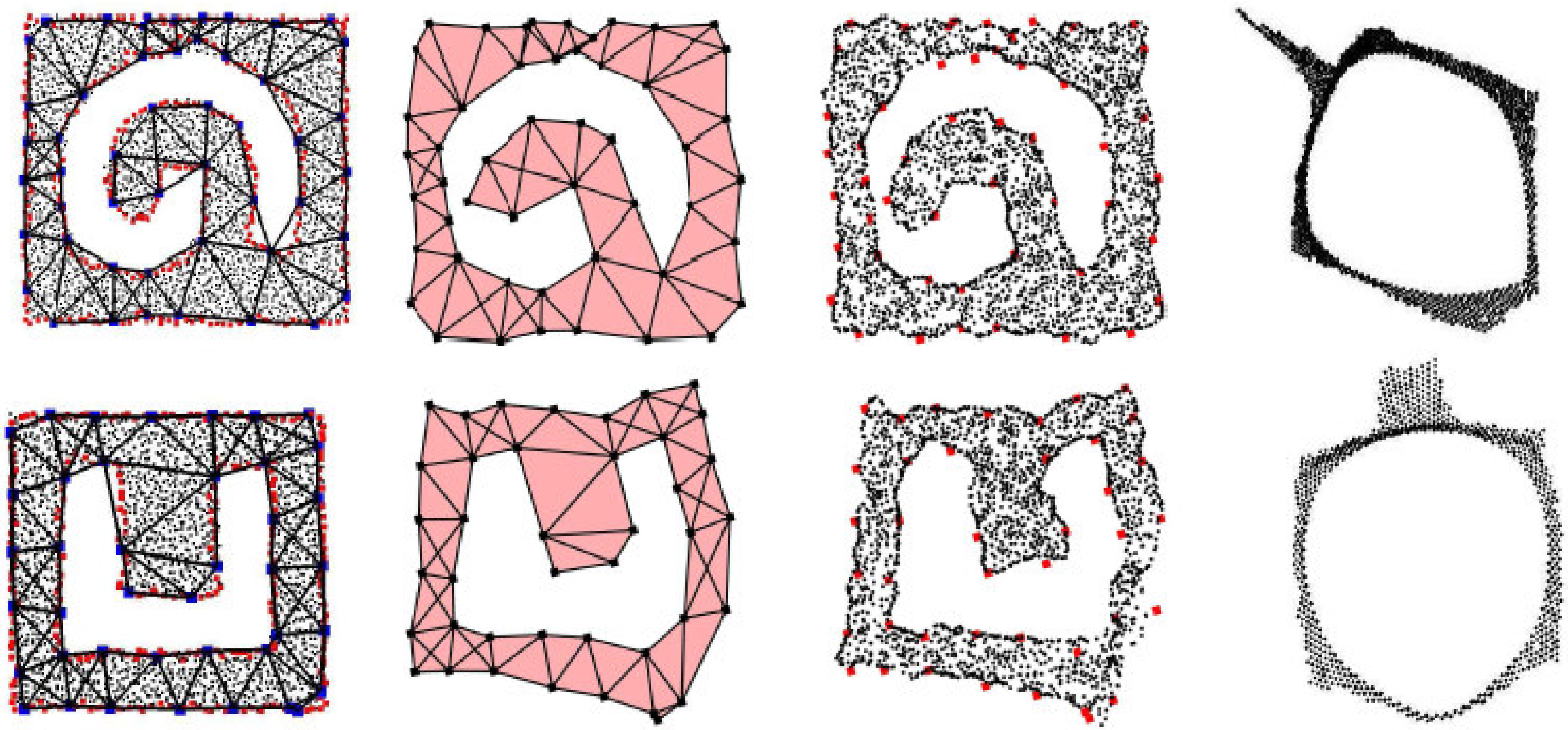

Unit disk graph model, Node average degree 6~10.

Conclusion

- Exploit the geometric and topological features of sensor networks.
- Simple algorithm, Surprising results.