

Connectivity-based Sensor Network Localization using Incremental Delaunay Refinement

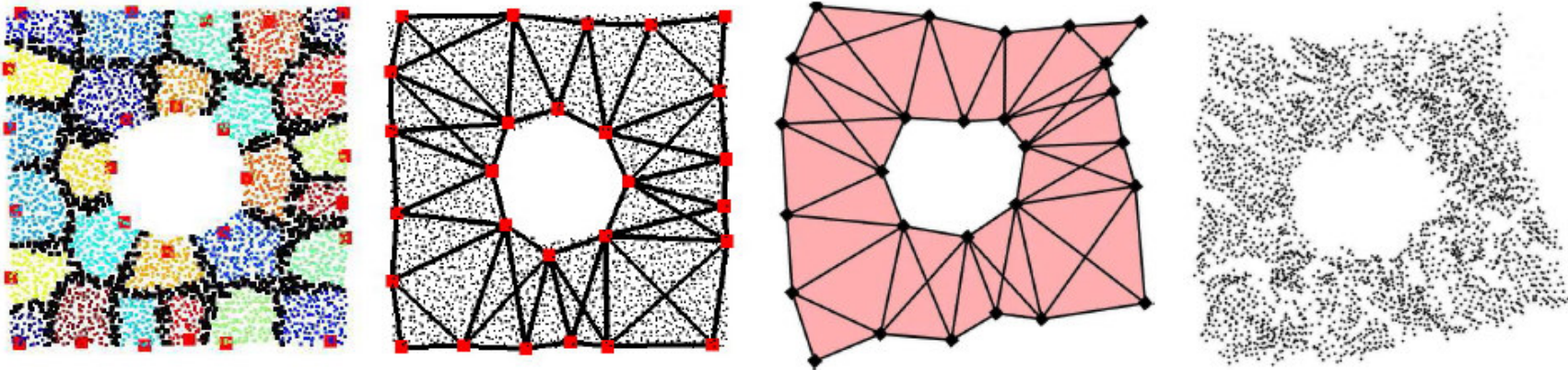
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Network localization problem

- Thousands of sensor nodes spread across some area
- Nodes talk to nearby neighbors
- Connectivity only, no special hardware, no range or angle info
- Complex geometry, multiple holes
- No anchor node

- Objective: recover the relative positions

Our solution: 5 steps to localization

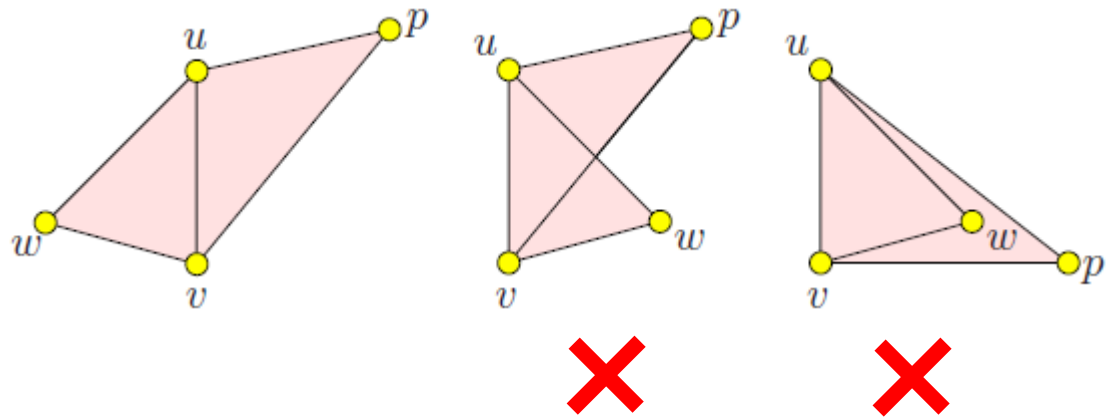
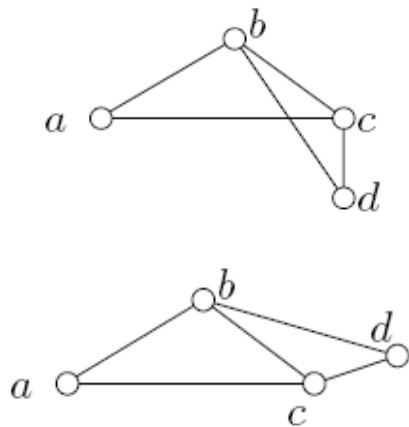


1. Select a small subset as **landmark** nodes.
2. Partition network into **Voronoi cells** based on hop count distance to landmark nodes
3. Extract dual **Delaunay complex**
4. Embed Delaunay complex (landmarks)
5. Localize all nodes based on distances to landmarks

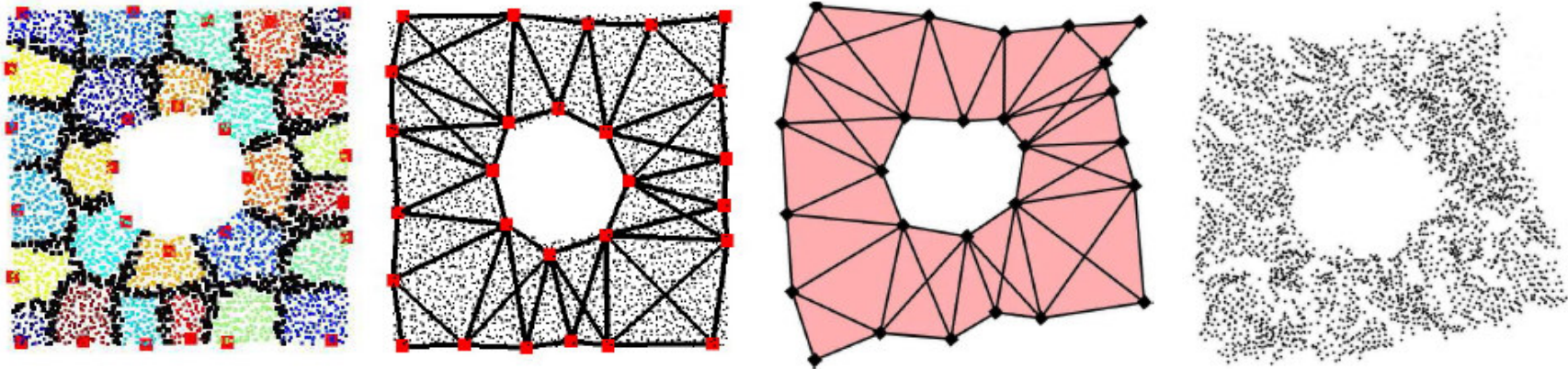
Key idea



- Embedding of the Delaunay complex avoids flip ambiguity—major obstacle in localization.
- Two Delaunay triangles must be put side by side.



This paper: landmark selection



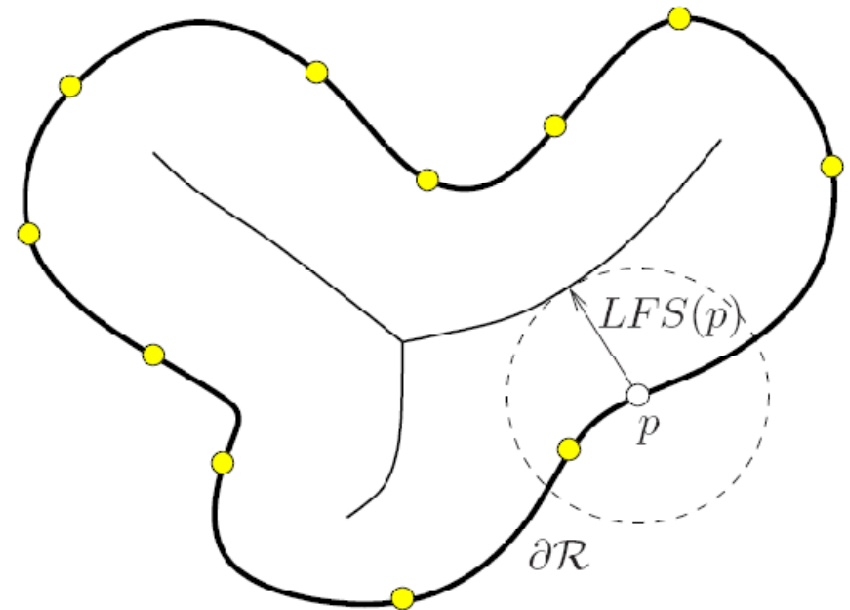
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Previous landmark selection scheme

1. Detect network boundary
2. Select nodes on the boundary with density \sim local geometric complexity

Successful boundary detection

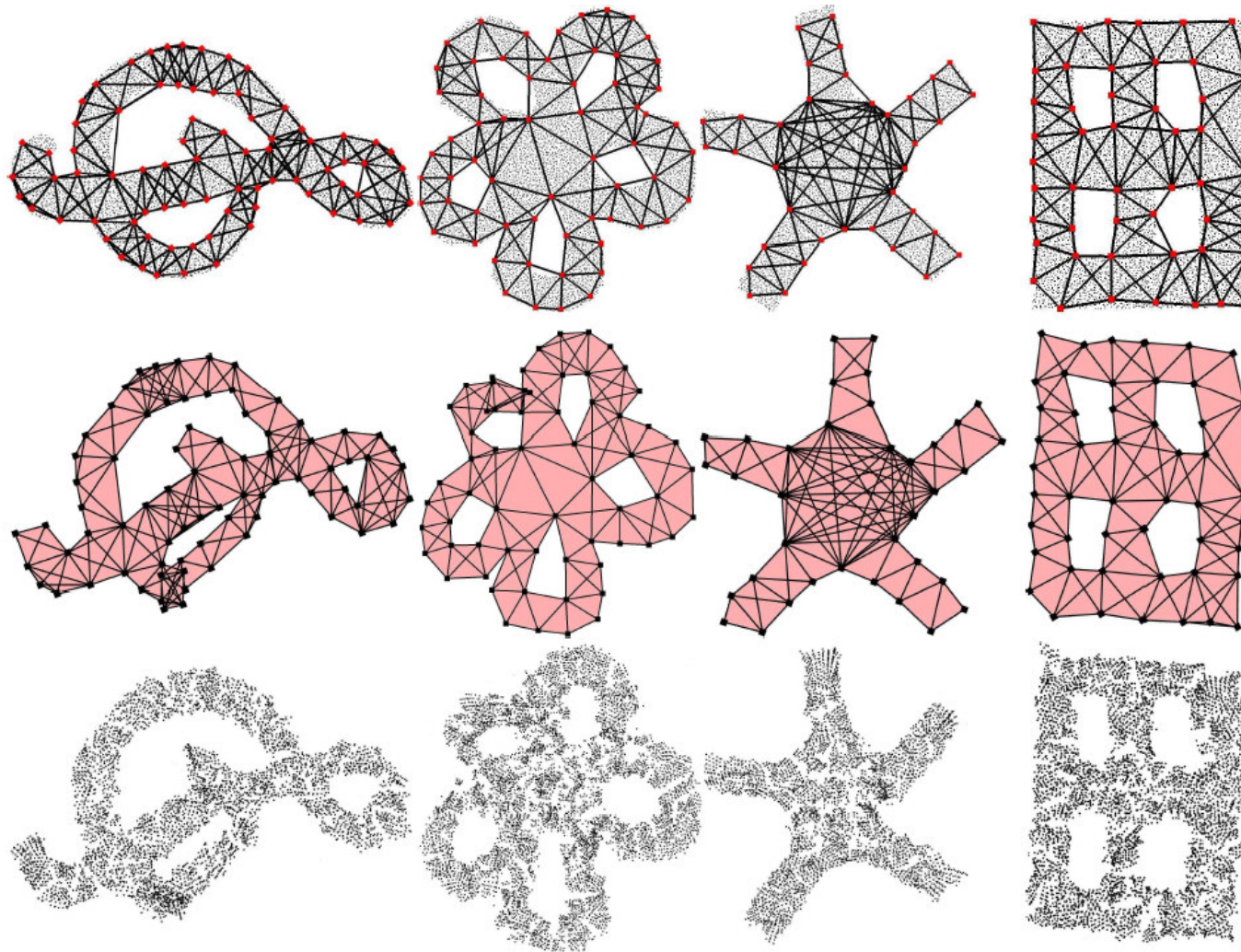
1. Require relatively high sensor density.
2. Network shape cannot be too complicated.



Incremental landmark selection

- No need to detect network boundary
- Allow parallel & distributed execution.
- Greatly improve algorithm robustness and applicability

Our result

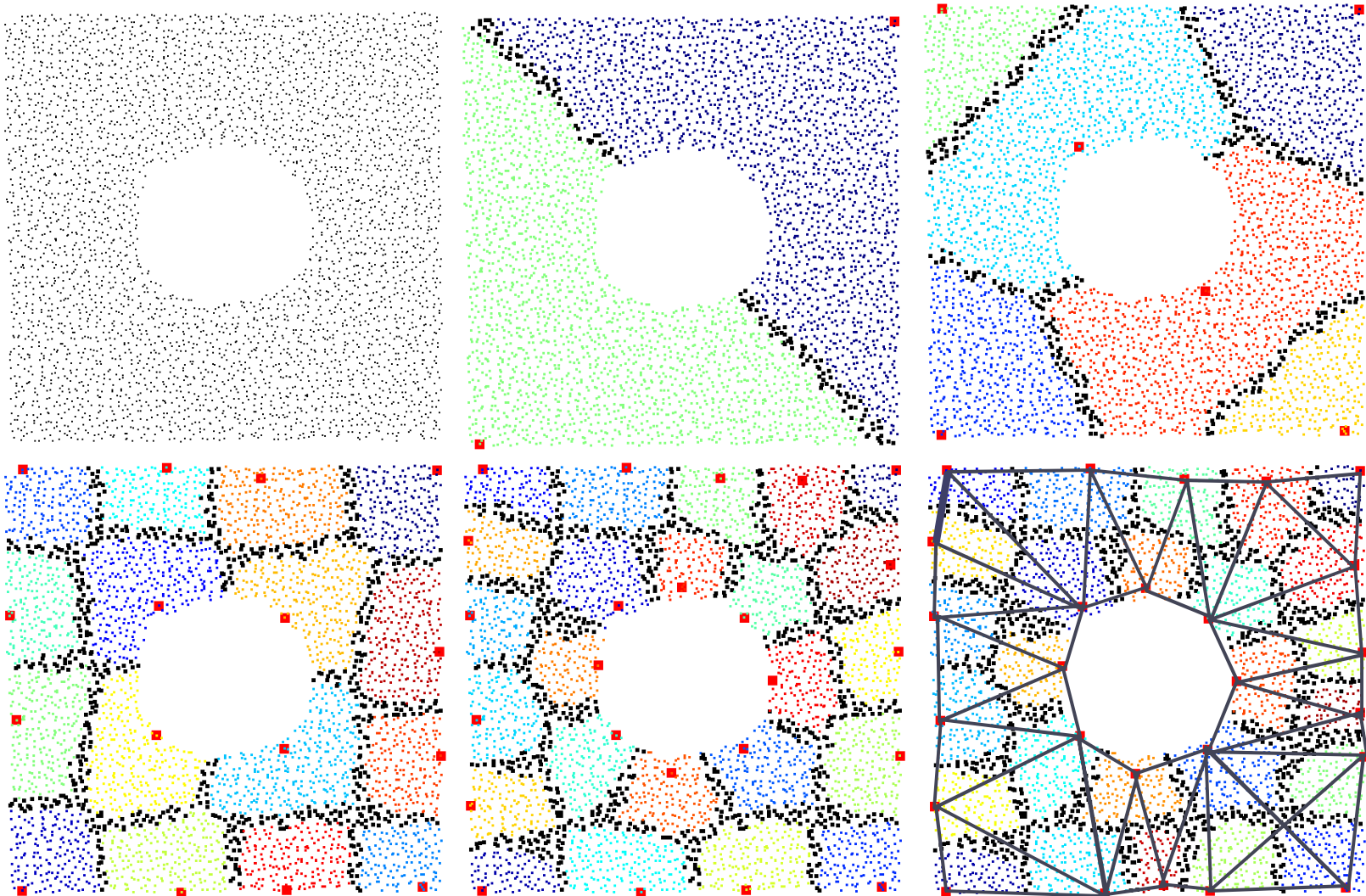


Ground
Truth

Embedding
Delaunay
Complex


Embedding
All Nodes

Incremental Construction of Voronoi Diagram



Outline of what's next...

1. Criteria we want to meet

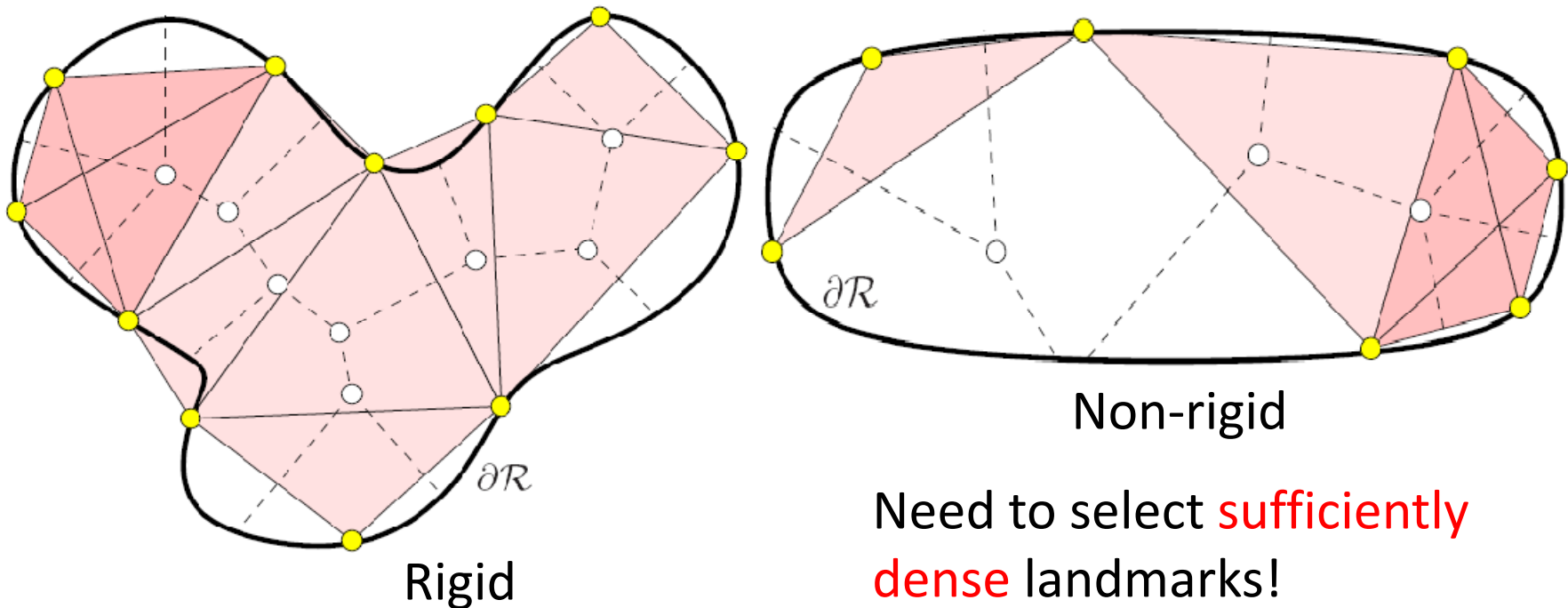
- a. Define parameters/definitions to capture progress toward reaching criteria
- b. Thm. that proves parameters do indeed measure progress toward goal 

2. Local graph conditions to address that imply criteria in 1 are being satisfied

3. Decentralized algorithm that achieves conditions in 2

#1: Criteria for good landmark selection (1 of 2)

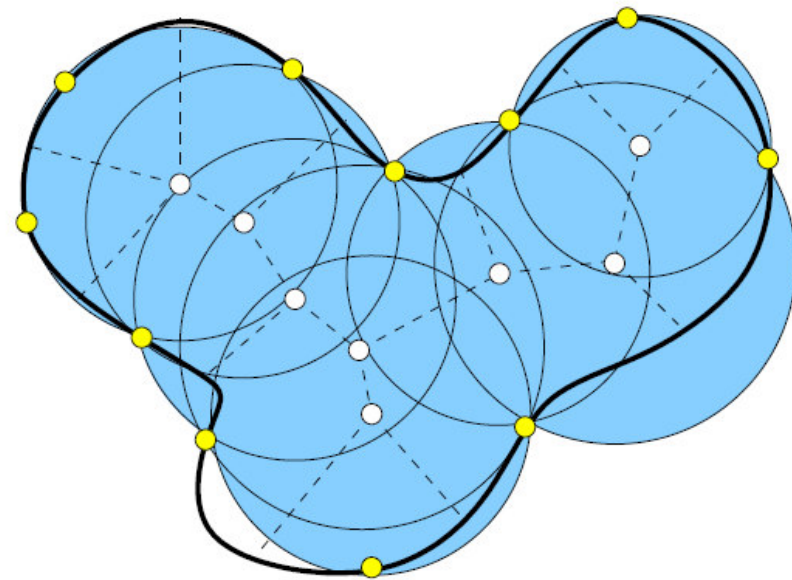
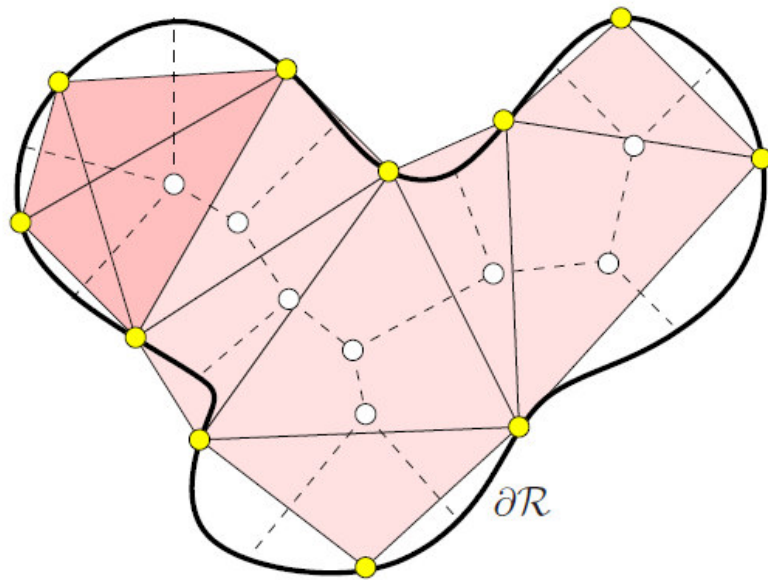
1. Global rigidity — Only one realization of Delaunay complex (no flip ambiguities!) A graph is rigid if one cannot deform the shape without changing the edge lengths.



#1: Criteria for good landmark selection (2 of 2)

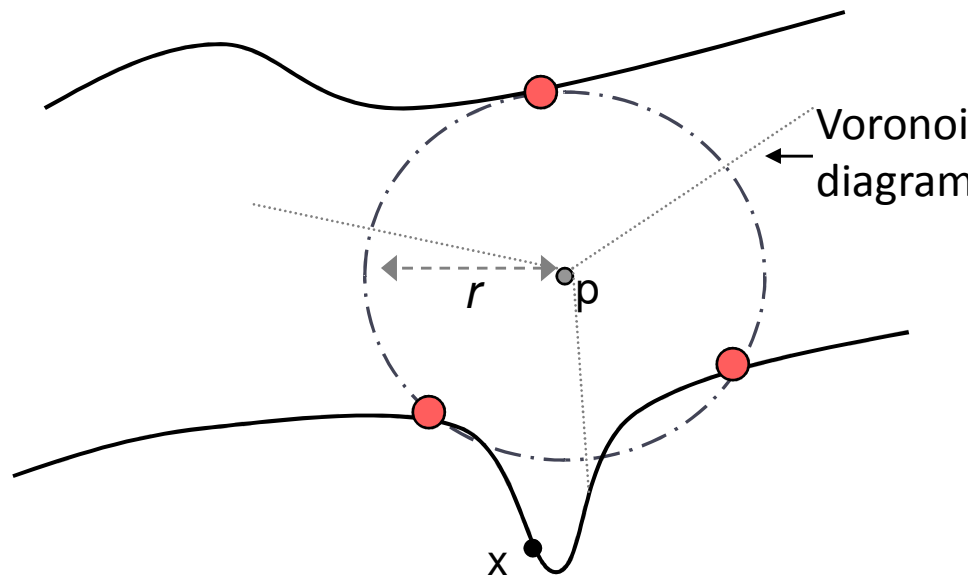
2. Coverage — no node should be very far from the Delaunay complex (i.e. Delaunay 'skeleton' should match network layout)

i.e. no node should be far outside Delaunay ball



Coverage

- **δ -coverage:** Delaunay complex δ -covers R if every point is within distance $(1 + \delta)r$ from the center p of a Voronoi ball $B_r(p)$, where r is the radius of the ball.



If $\delta = 1$,
then x cannot be
more than $2r$ from p

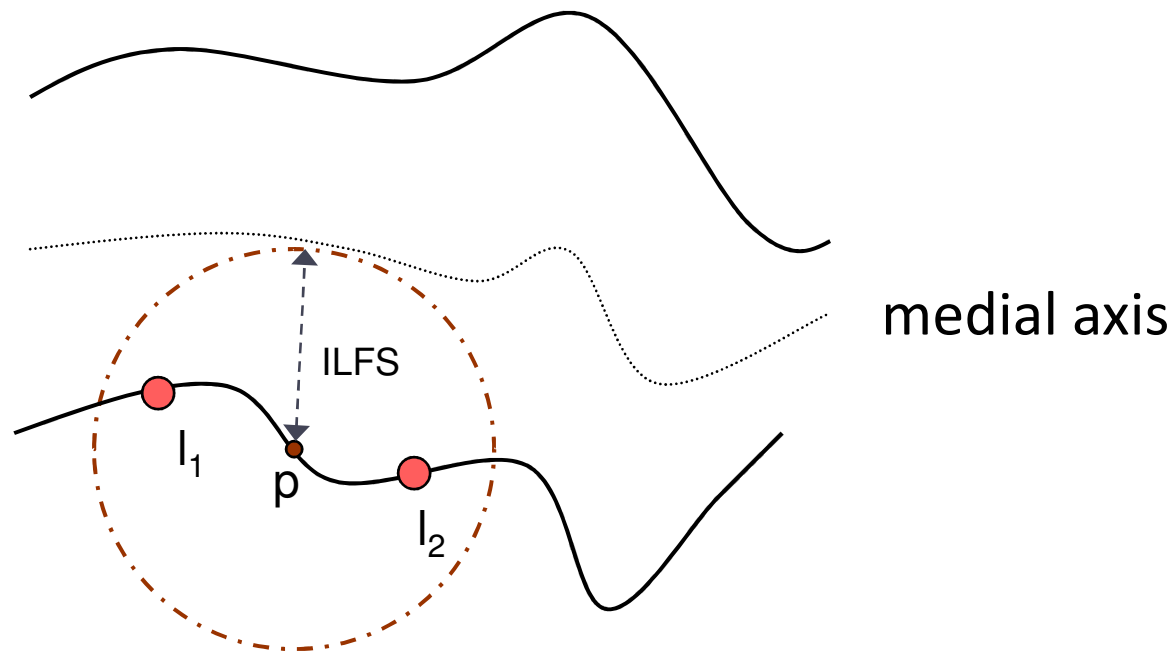
Our previous algorithm

γ -sample: For any point p on the boundary of R there is at least one landmark within distance $\gamma \cdot \text{ILFS}(p)$ from p .

Medial axis := $\{x \mid x \text{ has } \geq 2 \text{ closest pts on the boundary}\}$

$\text{ILFS}(p)$:= distance from p to the medial axis.

Alg: Select a **dense sample** (γ -sample for $\gamma < 1$).



Dense sample allows both rigidity and coverage

For a γ -sample with $\gamma < 1$:

- Theorem (INFOCOM'08): The Delaunay complex is rigid.
- Theorem (this paper): The Delaunay complex achieves δ -coverage, with $\delta = 2\gamma / (1 - \gamma)$.

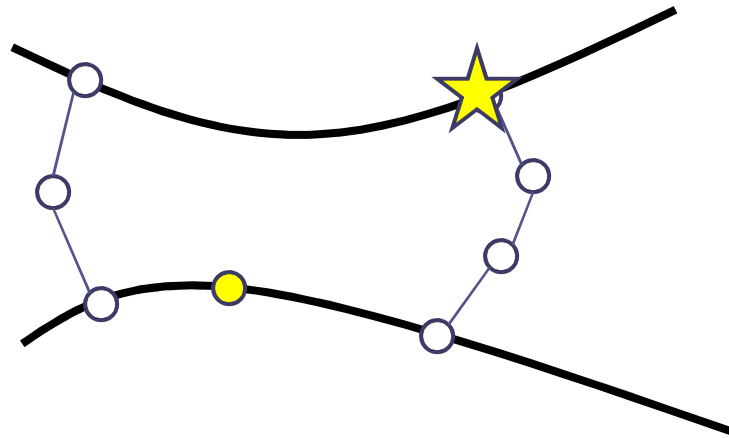
#2: Local graph conditions to satisfy

- 1. Local Voronoi edge connectivity:** The Voronoi edges for each landmark u form a connected set.
→ guarantees rigidity
- 2. Local Voronoi ball coverage:** Each node x inside a Voronoi cell $V(u)$ is δ -covered by a Voronoi ball $B_r(p)$, where p is a Voronoi vertex with landmark u .
→ implies coverage

#3: Algorithm to achieve conditions

For each Voronoi cell $V(u)$

1. If 1st condition not met, select among **Voronoi edge endpoints** the one **furthest** from u as a new landmark.

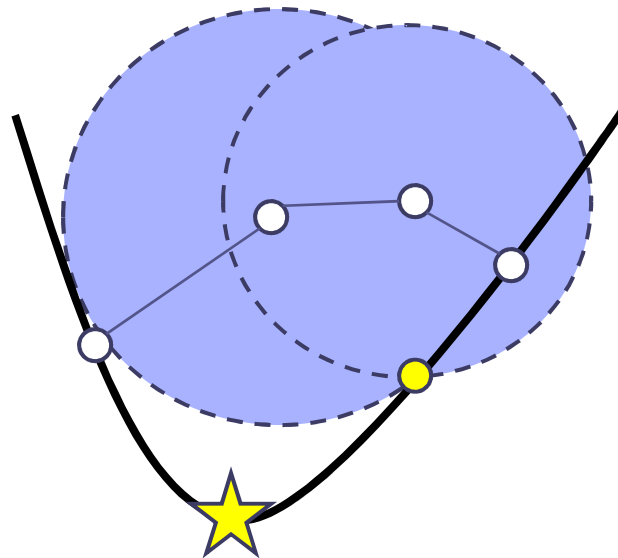


#3: Algorithm to achieve conditions

For each Voronoi cell $V(u)$

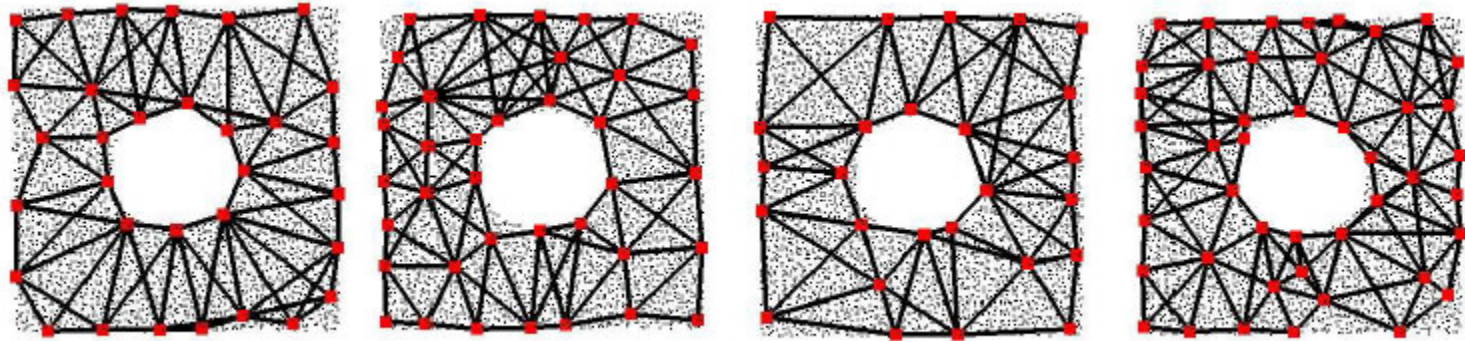
2. Then, if 2nd not met, select from among all points that violate coverage condition the one that is **least covered** as a new landmark.

$$\max_x \min_{B_r(p)} \{ \delta' \mid d(x, p) = (1 + \delta')r \}$$

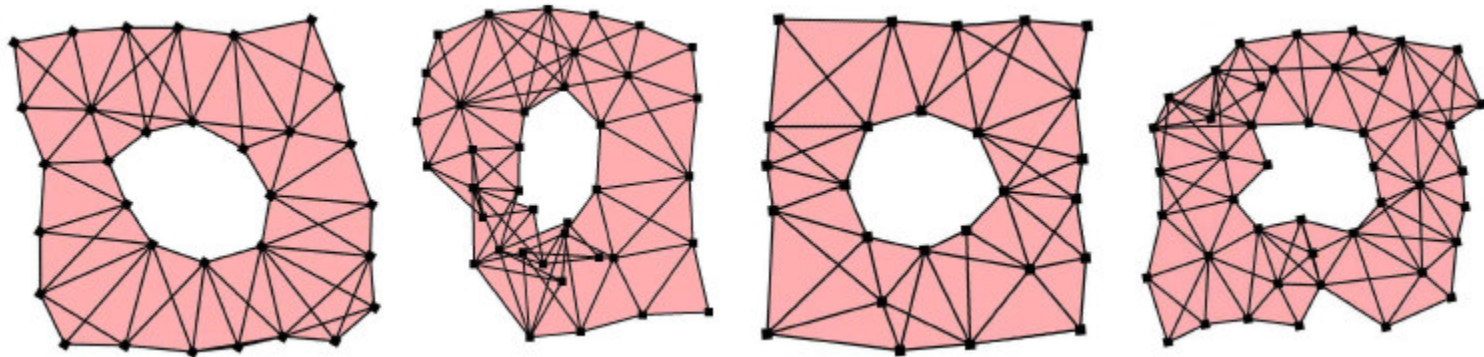


Simulations – effect of network model

Ground truth



Embedded landmarks



$\alpha = 0.6$
avg. deg. 6.4

$\alpha = 0.6$
avg. deg. 5.6

$\beta = 0.6$
avg. deg. 6.2

$\beta = 0.6$
avg. deg. 5.0

Quasi-UDG

Delete each edge with prob. $1 - \beta$

Improvement over previous algorithm

Previous algorithm dependant on efficacy of boundary detection algorithm

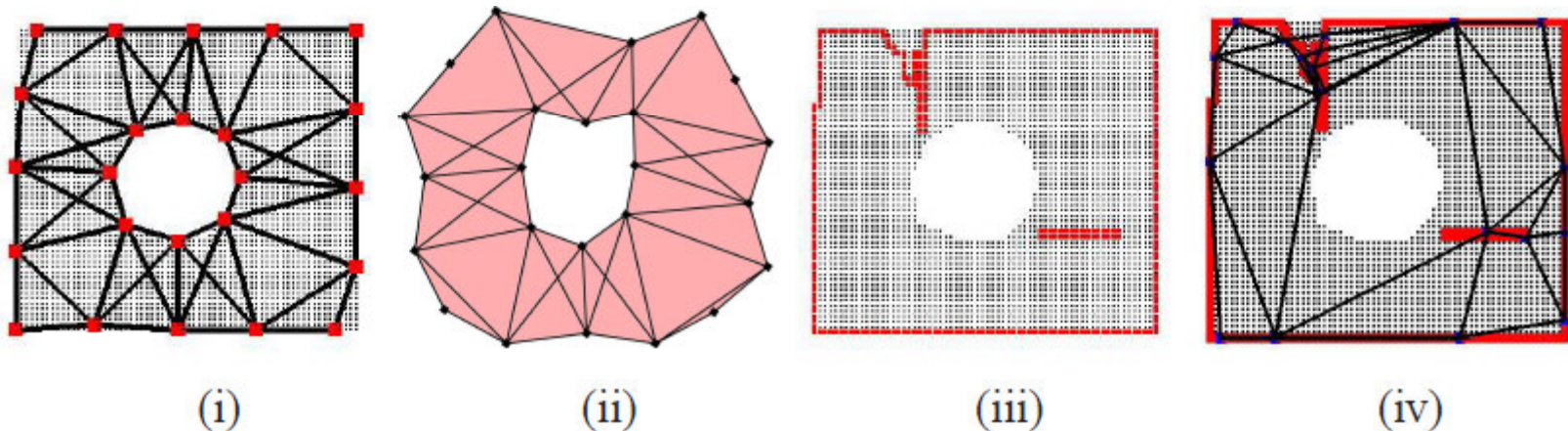
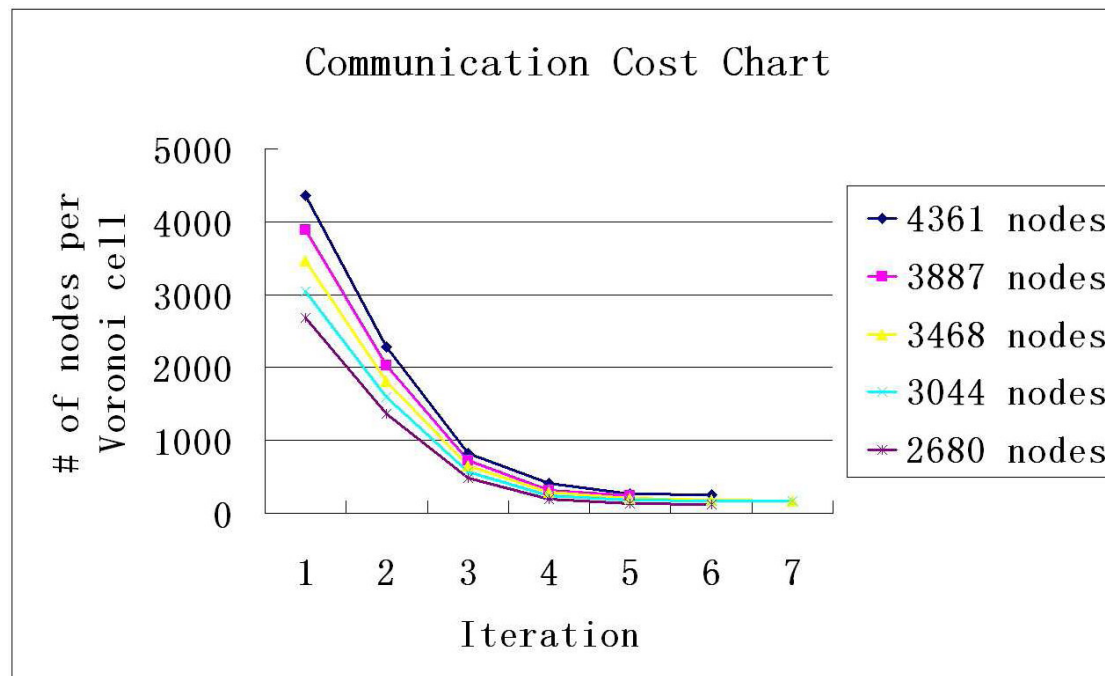


Fig. 9. A perfect grid network. 3388 nodes, avg. degree 3.87. (i) the Delaunay complex extracted from the Voronoi cells of the landmarks using the new algorithm. (ii) the embedding result. (iii) the boundary detection result. (iv) the Delaunay complex result using the previous algorithm.

Communication cost

- Grows slowly with # nodes.
- # iterations is small
- The size of Voronoi cell decreases quickly



Conclusion

With local landmark selection:

- localization algorithm is more robust
- less sensitive to the noisy results of boundary detection
- avoids its high computation cost

Questions/comments?

