Tradeoffs between Stretch Factor and Load Balancing Ratio in Routing on Growth Restricted Graphs

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Short paths and load balancing

A routing algorithm delivers a packet with size ℓ_i from source s_i to destination t_i.

Two desirable properties on routing algorithms:

Short paths: e.g. shortest path routing.

Load balancing: nodes are evenly loaded.

Stretch factor and load balancing ratio

Stretch factor: $\max\{\frac{\text{routing path length}}{\text{shortest path length}}\}$;

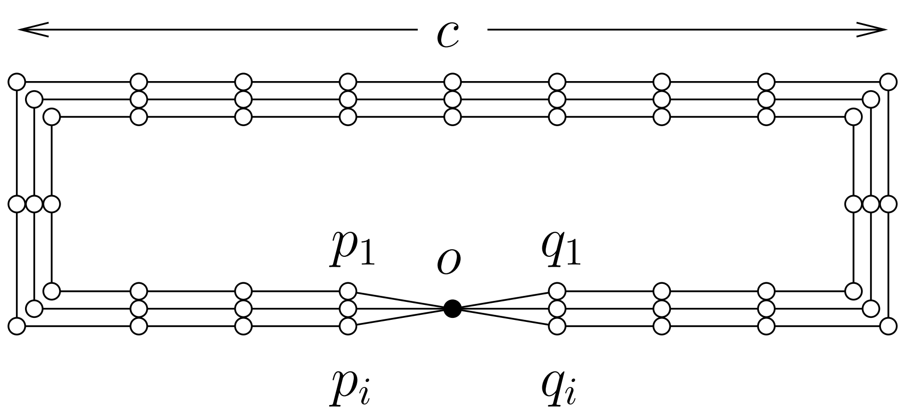
A path is c-short if it's length is at most c times the shortest path length.

Load balancing ratio: $\frac{\text{max traffic on any node}}{\text{max traffic of the OPT off-line Alg}}$.

Question: Can we have good ratios under both measures simultaneously?

For general graphs, the answer is No

These two properties are conflicting to some extent.



All the c-short paths from p_i to q_i have to pass though o. So the load balancing ratio for c-short routing is $\Omega(n/c)$.

Growth-restricted graphs

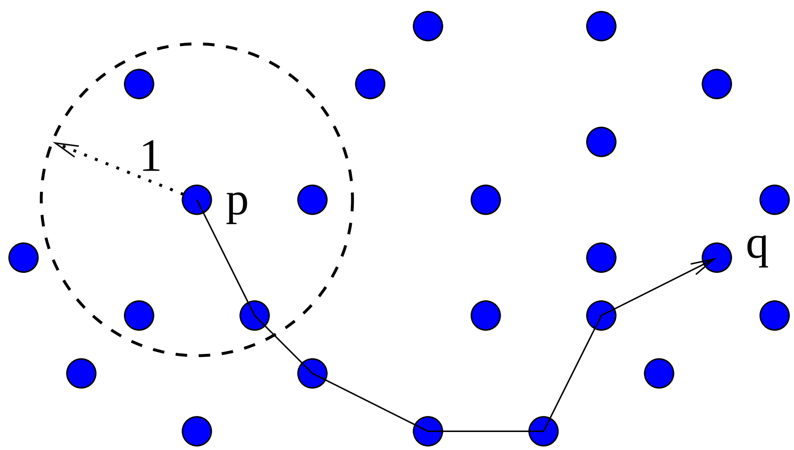
A graph has growth rate k and density ρ if the number of nodes within distance r of any vertex is no more than ρr^k.

Graphs with restricted growth appear naturally:

- Meshes, VLSI layout networks.
- Internet latencies embedded in geometric space, e.g., [Ng&Zhang 2002];
- Peer-to-peer overlay networks;
- Wireless ad hoc networks.

Ad hoc wireless networks

Unit-disk Graphs: A set of wireless nodes in the plane, any two can communicate directly if they are within Euclidean distance 1.



If "node density" (# nodes inside any unit disk) $\leq \rho$, then a disk with radius r can only cover $O(\rho r^2)$ nodes – the unit disk graph has density $O(\rho)$ and growth rate 2.

Results

For a graph with density ρ and growth rate k, the load balancing ratio of the optimal c-short routing is $\Theta((n/c)^{1-1/k}\rho^{1/k})$.

Shortest path routing on unit disk graphs with constant density, or 2-d meshes has load balancing ratio $\Theta(\sqrt{n})$.

An online algorithm that produces $\Theta(\log n)$ approximate load balancing ratio by using only c-short paths.

For unit disk graphs, if the average node density is $\overline{\rho}$, the load balancing ratio of shortest path routing is $O(\min(\sqrt{\overline{\rho}n}\log n, n))$.

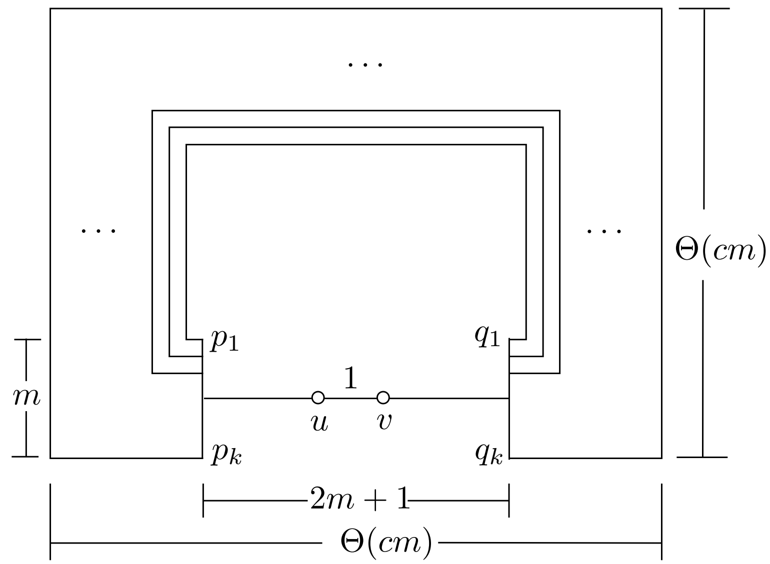
The load balancing ratio of OPT routing on a c-spanner of a unit disk graph is $\Theta(\rho r^2)$.

This talk

For a unit disk graph with maximum node density ρ, assume the optimal load balanced routing algorithm produces maximum load ℓ^*, shortest path routing produces maximum load ℓ^1.

Theorem 0.1. *The load balancing ratio of shortest path routing is* $\ell^1/\ell^* = \Theta(\sqrt{\rho n})$.

Lower bound



Choose $m = \sqrt{n/(c\rho)}$. The load balancing ratio of any c-short routing is $\Omega(\rho m) = \Omega(\sqrt{n\rho/c})$.

Upper bound proof

Assume the requests are $R = \{(s_i, t_i, \ell_i)\}$ and p is the node with highest load by shortest path routing.

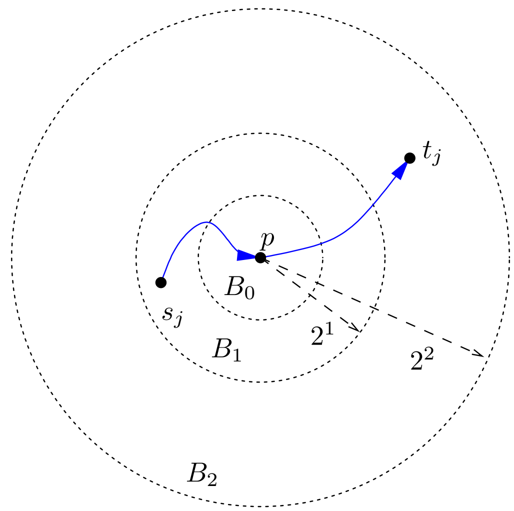
W.l.o.g, assume all the requests R are routed through p by shortest path routing. $\ell^1 = \ell(p) = \ell = \sum \ell_i$.

We want to upper bound $\alpha = \ell / \ell^*$.

Intuition: if shortest path routing creates a high load on p, the OPT algorithm also has to create a high load.

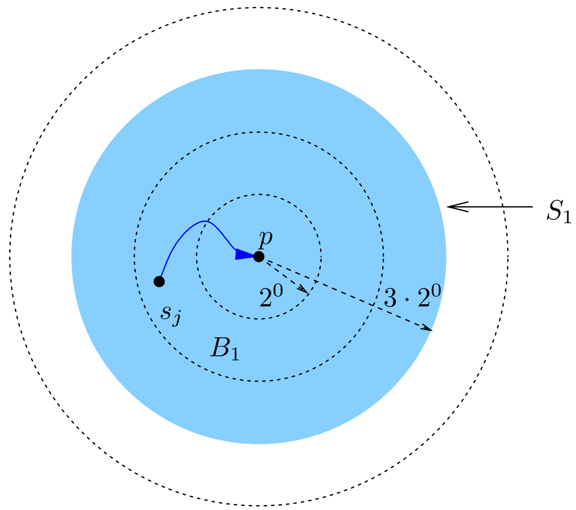
Upper bound proof, cont.

Now we study how to route the requests R_i generated in each annulus B_i (with inner radius 2^{i-1} and outer radius 2^i) by the OPT routing algorithm.



The shortest path from s_j to t_j goes through p. So the optimal routing path P_j has length at least 2^{i-1}. Take A_j as the first 2^{i-1} nodes on P_j. $S_i = \bigcup_{s_j \in B_i} A_j$.

Upper bound proof, cont.



The total load on S_i: $\sum_{v \in S_i} \ell(v) \geq \sum_{s_j \in B_i} \ell_j |A_j| = 2^{i-1} \sum_{s_j \in B_i} \ell_j$.

On the other hand, each node has maximum load ℓ^*.
$\sum_{v \in S_i} \ell(v) \leq |S_i| \ell^* \leq c_0 \rho (2^{i-1})^2 \ell / \alpha$.

Therefore, $\sum_{s_j \in B_i} \ell_j / \ell \leq c_0 \rho 2^{i-1} / \alpha$.

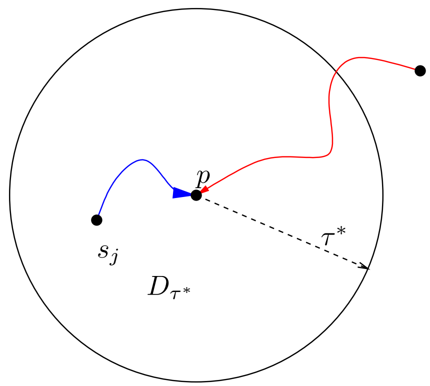
Upper bound proof, cont.

For each B_i, $\sum_{s_j \in B_i} \ell_j/\ell \leq c_0 \rho 2^{i-1}/\alpha$.

By summing up over all i, we have,

Lemma 0.2. *Suppose D_τ is the disk with radius τ centered at p, then $\sum_{s_j \in D_\tau} \ell_j/\ell \leq c_0 \rho \tau/\alpha$, for some constant $c_0 > 0$.*

Upper bound proof, cont.



Take τ^* to be the smallest radius such that $\sum_{s_j \in D^*_\tau} \ell_j/\ell \geq 1/2$. Then $\alpha \leq 2c_0\rho\tau^*$.

On the other hand, for all the requests generated outside D_{τ^*}, each ℓ_i will contribute at least $\ell_i\tau^*$ to the total load. $\ell^* \geq \ell\tau^*/(6n)$.

Therefore, $\alpha \leq \min(2c_0\rho\tau^*, 6n/\tau^*) \leq O(\sqrt{\rho n})$. QED

Growth restricted graphs

The proof can be extended to show,

Theorem 0.3. *For a graph with density* ρ *and growth rate* k*, , the load balancing ratio of the optimal* c*-short path routing is* $\Theta((n/c)^{1-1/k}\rho^{1/k})$.

Short path load balanced routing

We propose an online algorithm with $\Theta(\log n)$-approximate load balancing ratio using only c-short paths.

Idea: Modify the online algorithm [Aspnes *et al.*, 1993] for competitive online virtual circuit routing.

Assign weights to edges.

Find the “lightest weight” path among all c-short paths.

Conclusion

Contribution: a thorough study on the tradeoffs between stretch factor and load balancing ratio for routing on growth restricted graphs.

- Tight bounds;
- An online algorithm with competitive load balancing ratio by using only short paths.

Thank You!