CSE540, Fall 2008

Homework 1

Basic Questions

B1. Consider the following TM M: M has states $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$, input alphabet $\Sigma = \{a, b\}$, tape alphabet $\Gamma = \{a, b, \mathtt{B}\}$, initial state q_0, final states $F = \{q_5\}$, and the transition function δ:

δ	a	b	$\mathtt{B}$
q_0	q_1, a, R	q_2, b, R	$q_3, \mathtt{B}, L$
q_1	q_1, a, R	q_2, a, R	$q_3, \mathtt{B}, L$
q_2	q_1, a, R	q_2, b, R	$q_3, \mathtt{B}, L$
q_3	q_4, a, L	$q_4, \mathtt{B}, L$	
q_4	q_4, a, L	q_4, b, L	$q_5, \mathtt{B}, R$

(a) Trace M on inputs *aaabba* and *bbbaba* (show the transitions of the machine configuration).

(b) What is the function computed by M (i.e., describe for an input string $w \in \{a, b\}^*$, what is the output)?

B2. Exercise 1.6 of the textbook.

B3. Exercise 1.12 of the textbook.

B4. Show that the function $f(n, m) = 2^n(2m + 1) - 1$ is a pairing function.

B5. Show that the set F of all one-to-one, increasing functions from $\mathbf{N}$ to $\mathbf{N}$ is uncountable. [Note: A function $f : \mathbf{N} \to \mathbf{N}$ is *increasing* if $f(k) \leq f(k+1)$ for all $k \in \mathbf{N}$.]

B6. Assume that $A, B, C \subseteq \{0, 1\}^*$ are r.e. sets, and assume that M_A, M_B and M_C accept sets A, B and C, respectively. Let

$$D = (A \cap B) \cup (B \cap C) \cup (C \cap A).$$

(a) Construct, by the dovetailing technique, a TM M_D that accepts D.

(b) Find a recursive predicate R_D such that $D = \{x \mid (\exists y)\ R_D(x, y)\}$.

B7. Prove that set $A = \{n \mid \{0, 1, \ldots, n\} \subseteq W_n\}$ is an r.e. set.

B8. Show that an infinite set A is a recursive set if and only if A is the range of a strictly increasing recursive function f.

Medium Questions

M1. Exercise 1.2 (a), (b) of the textbook ("Theory of Computational Complexity").

M2. Prove that set $B = \{n \mid |W_n \cap \{0, 1, \ldots, n\}| \geq n/2\}$ is an r.e. set.

M3. Prove that set $C = \{\langle n, x\rangle \mid$ there exist $n_1, n_2, \ldots, n_k$, for some $k \geq 1$, such that $x \in W_{n_1}$, $n_1 \in W_{n_2}, \ldots$, $n_{k-1} \in Wn_k, n_k \in W_n\}$ is an r.e. set.

M4. Assume that $A \subseteq \mathbf{N}$ is a set with the following properties:

(a) For all $n \in A$, ϕ_n is a recursive function (i.e., ϕ_n halts on all inputs), and

(b) For every recursive function f (that halts on all inputs), $f = \phi_n$ for some $n \in A$.

Prove that A is not an r.e. set. [Hint: by diagonalization.]

M5. Show that the set $D = \{\langle i, j\rangle \mid W_i = \overline{W_j}\}$ is not an r.e. set.