6.034 Recitation 6: Constraint Satisfaction Problem Solution (10/21/05)

LOrtiz (Orig. by KKoile)

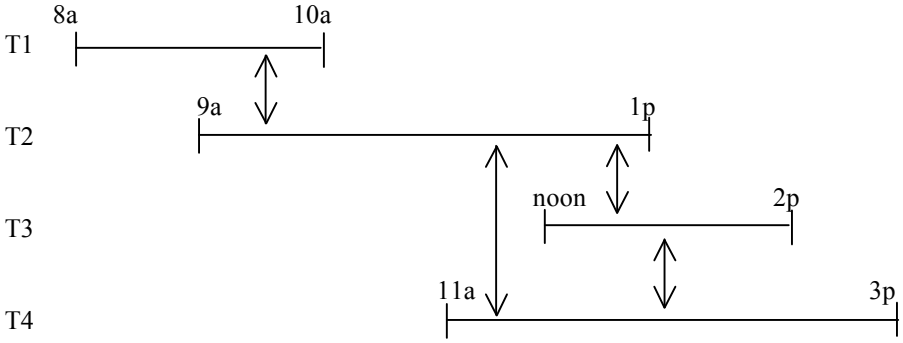
Let's look at a train scheduling problem similar to the one mentioned in lecture.

There are 4 trains (T1 T2 T3 T4) and three locomotives.(L1 L2 L3). What assignments of locomotives to trains will satisfy the following schedule:

Train	in use
T1	8am to 10am
T2	9am to 1pm
T3	noon to 2pm
T4	11am to 3pm

Make the following assumptions:
Each locomotive can only pull one train at a time.
Each locomotive has time to get to the station where its next train is located.
L3 is too small to pull T3.
L2 and L3 are too small to pull T4.

1. Draw timelines representing when each train is in use, and show constraints by drawing arrows between timelines representing trains that cannot use the same locomotive.



2. What are the variables? trains T1 T2 T3 T4

What are the values? locomotives L1 L2 L3

What are the possible values for each variable?

train	locomotives
T1	L1 L2 L3
T2	L1 L2 L3
T3	L1 L2
T4	L1

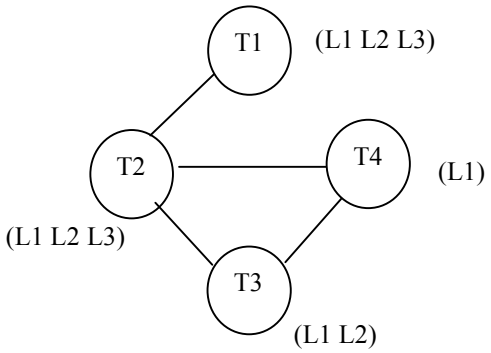
Now reverse your choice of variables and values. What would the possible values be for those variables?

With locomotives as variables and trains as values, the possible values are sets of trains. In this case, since there are 3 locomotives and 4 trains, 1 locomotive is going to have to pull 2 trains. So the possible values are going to be the subsets of trains of size 1 and size 2.

Alternatively, if we say that each locomotive could pull 2 trains, we could have variables for each instance of a locomotive pulling a train. So we would have L1-1, L1-2, L2-1, L2-2, etc. The possible values would be the set of trains that could be pulled, plus no train. The constraints would be that two instances can't pull the same train, and two instances of the same locomotive can't pull trains that overlap in time.

The message here: There is often a natural way to represent the variables and values that makes the problem much easier to formulate!

3. Draw the constraint diagram for this problem. assuming trains as variables and locomotives as values. Label each node with the variable it represents and the domain for that variable. What do the links represent?



The links represent the constraint that no trains that overlap in time can be pulled by the same locomotive.

4. Try finding a solution to this problem using only constraint propagation. Finish filling in the table below (as in the online recitation notes). Are the arcs consistent at the end of the propagation procedure? Why is constraint propagation in general not sufficient to solve constraint satisfaction problems?

arc	value deleted
T1-T2	none
T2-T3	none
T2-T4	T2(L1)
T3-T4	T3(L1)
repeat for vars whose domain changed: T2, T3	
T2-T1	none
T2-T3	T2(L2)
T2-T4	none
T3-T4	none
repeat for vars whose domain changed: T2	
T2-T1	T1(L3)
T2-T3	none
T2-T4	none
repeat for vars whose domain changed: T1	
T1-T2	none

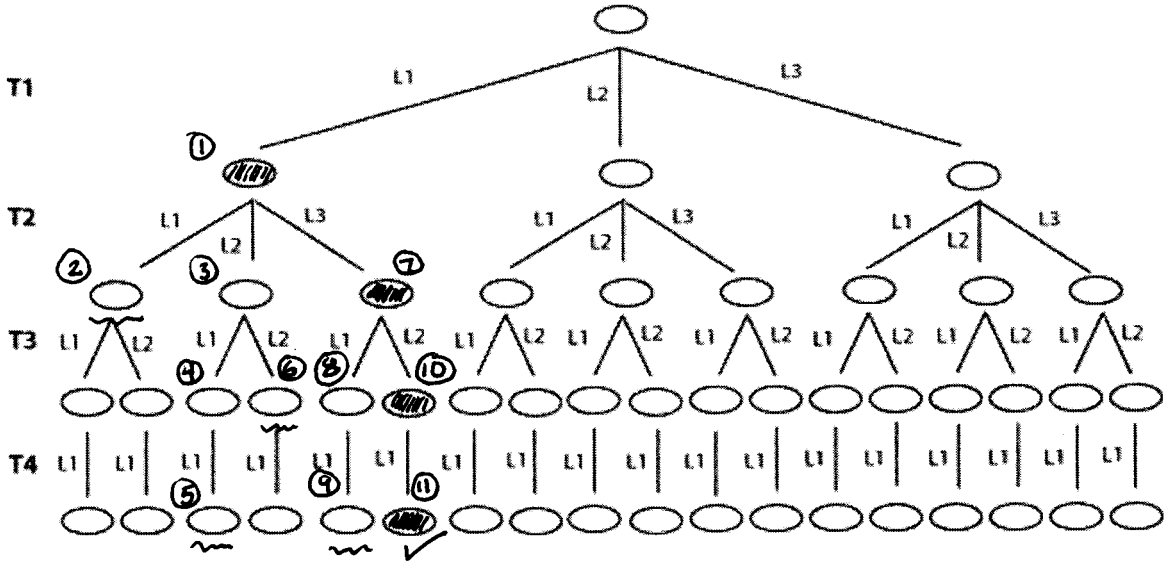
The arcs are consistent, and the variables have the following domain values:
T1 (L1 L2)
T2 L3
T3 L2
T4 L1

In general, constraint propagation isn't sufficient to solve constraint satisfaction problems because there still may be any number of solutions, including zero. (In this case, we can see that there are two solutions, but in general, the solutions may not be so obvious.)

5. Given the search tree below, find a solution using backtracking (without forward checking). (Note that we're assuming variable assignment order of T1, T2, T3, T4, and value assignment order of L1, L2, L3.) Show assignments of values to variables in order by labeling arcs on the graph in consecutive order. (Assume assignments are made, then checked for validity.)

How many assignments were made? 11

What is the solution? T1 = L1, T2 = L3, T4 = L1



6. Now find a solution using backtracking with forward checking. Label the tree as in the previous question, showing assignments of values in order. Also show which branches are pruned by forward checking. (Draw a line across the arc with the disallowed value.)

How many assignments were made? 7

What is the solution? (same as above)