

Practice Problems for Constraint Satisfaction Problems

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1. (6.034 Spring 2005 Quiz 1 Problem 2, Modified ¹) Consider assigning colors to a checkerboard so that squares that are adjacent vertically or horizontally do not have the same color. We know that this can be done with only two colors, say red (R) and black (B). We will limit our discussion to **five squares** on a 3x3 board, numbered as follows:

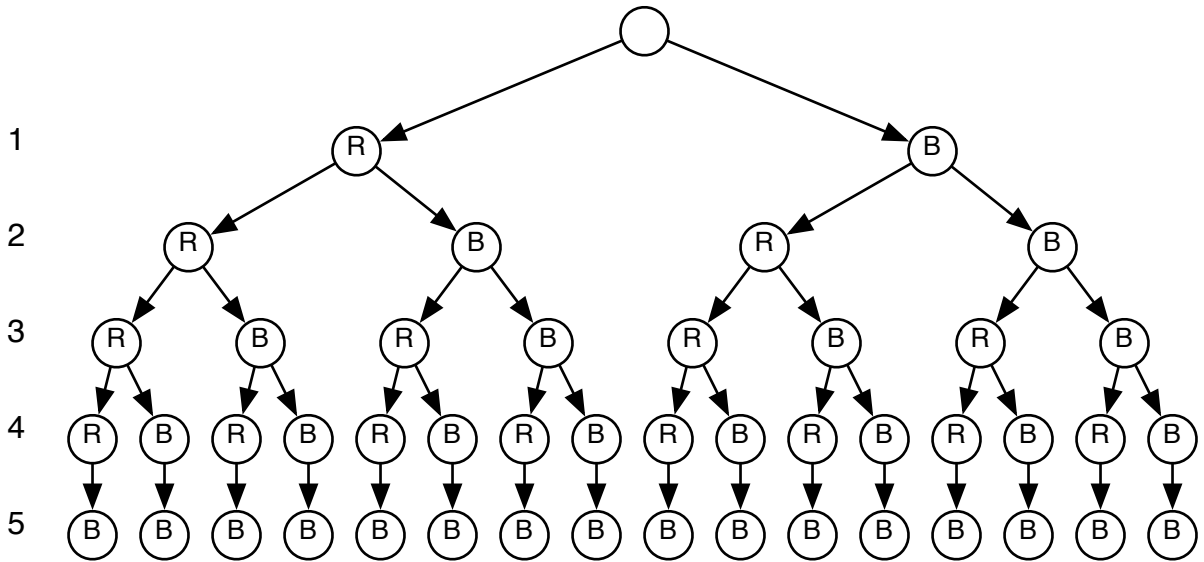
```
  1 | 2 | 3
  ---
  4 | 5 |
  ---
    |   |
```

Let's look at the CSP formulation of this problem. Let the squares be the variables and the colors be the values. All the variables have domains { R, B }.

- (a) If we run full constraint propagation on the initial state, what are the resulting domains of the variables?
- (b) Say, instead, the initial domain of variable 5 is restricted to { B }, with the other domains as before. If we now run full constraint propagation, what are the resulting domains of the variables?
- (c) If in the initial state (all variables have domains { R, B }), we assign variable 1 to R and do forward checking, what are the resulting domains of the other variables?

¹Only the last part was modified.

- (d) Assume that during backtracking we first attempt assigning variables to R and then to B. Assume, also, that we examine the variables in numerical order, starting with 1. Also, let the domain of variable 5 be { B }, the other domains are { R, B }. In the following tree, which shows the space of assignments to the 5 variables we care about, indicate how pure backtracking (BT) would proceed by placing a check mark next to any assignment that would be attempted during the search and crossing out the nodes where a constraint test would fail. Leave unmarked those nodes that would never be explored.



- (e) If we use backtracking with forward checking (BT-FC) in this same situation, give a list of all the assignments attempted, in sequence. Use the notation variable = color for assignments, for example, 1=R.
- (f) If we use backtracking with forward checking (BT-FC) and propagating through reduced singleton domains, give a list of all the variable assignments attempted, in sequence. Use the notation variable = color for assignments, for example, 1=R.

2. **(6.034 Spring 2005 Quiz 1 Problem 3)** You are trying to schedule observations on the space telescope. We have m scientists who have each submitted a list of n telescope observations they would like to make. An observation is specified by a target, a telescope instrument, and a time slot. Each scientist is working on a different project so the targets in each scientist's observations are different from those of other scientists. There are k total time slots, and the telescope has three instruments, but all must be aimed at the same target at the same time.

The greedy scientists cannot all be satisfied, so we will try to find a schedule that satisfies the following constraints:

- C1.** Exactly two observations from each scientist's list will be made (the choice of the two will be part of the solution).
- C2.** At most one observation per instrument per time slot is scheduled.
- C3.** The observations scheduled for a single time slot must have the same target.

Note that for some set of requested observations, there may not be any consistent schedule, but that's fine.

Consider the following three formulations of the problem.

- A.** The variables are the $3k$ instrument/time slots.
- B.** The variables are the m scientists.
- C.** The variables are the mn scientists' requests.

For each formulation, specify

1. The value domain for the variables.
2. The size of the domain for the variables (in terms of k , m , and n).
3. Which of the constraints are necessarily satisfied because of the formulation.
4. Whether the constraints can be specified as binary constraints in this formulation. If they can, explain how. If not, provide a counterexample.

Formulation A: The variables are the $3k$ instrument/time slots.

1. Domain:
2. Size of domain:
3. Satisfied constraints:
4. Binary constraints?:

Formulation B: The variables are the m scientists.

1. Domain:
2. Size of domain:
3. Satisfied constraints:
4. Binary constraints?:

Formulation C: The variables are the mn scientists' requests.

1. Domain:
2. Size of domain:
3. Satisfied constraints:
4. Binary constraints?: