

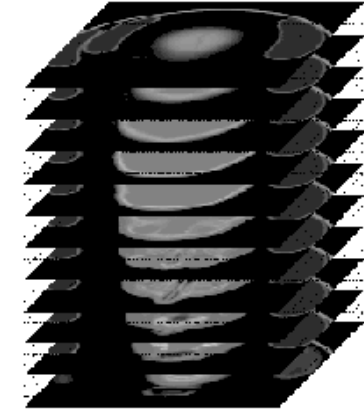
# Generation of Volumetric Data - Overview

- Medicine: imaging methods such as CT, MRI, fMRI, PET, SPECT, ultrasound
- Science and engineering:
  - simulations: flow, stress, heat, nuclear activity
  - observations: wind, weather, pressure, satellite
  - design: CAD / CAM
- Geology:
  - explorations: seismology, oil, precious metals
  - map making: satellite terrain mapping (also defense)
- Biology and pharmacology:
  - imaging: confocal microscopy
  - modeling: molecular bindings, molecule docking
- Security: luggage scanners at airports
- Entertainment: games, special effects, weather forecast
- Industry: reverse engineering and quality control with industrial CT

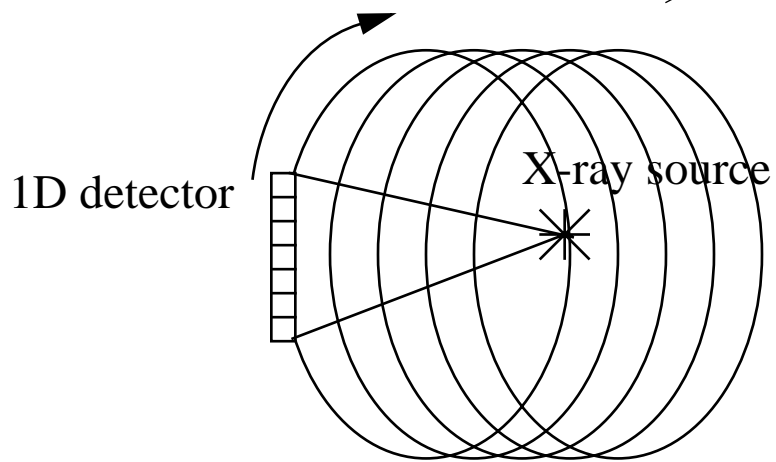
# Computed Tomography (CT) - Overview



3D reconstruction

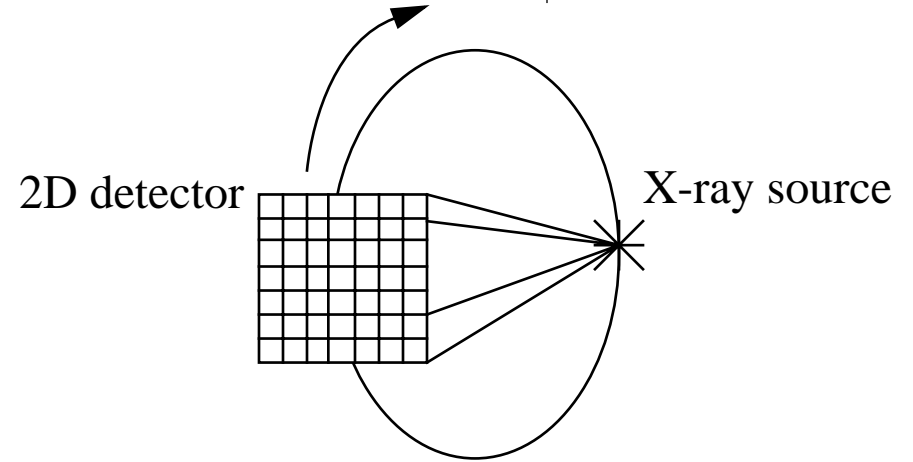


Scanning geometries:



circle-and-advance

helical (spiral)

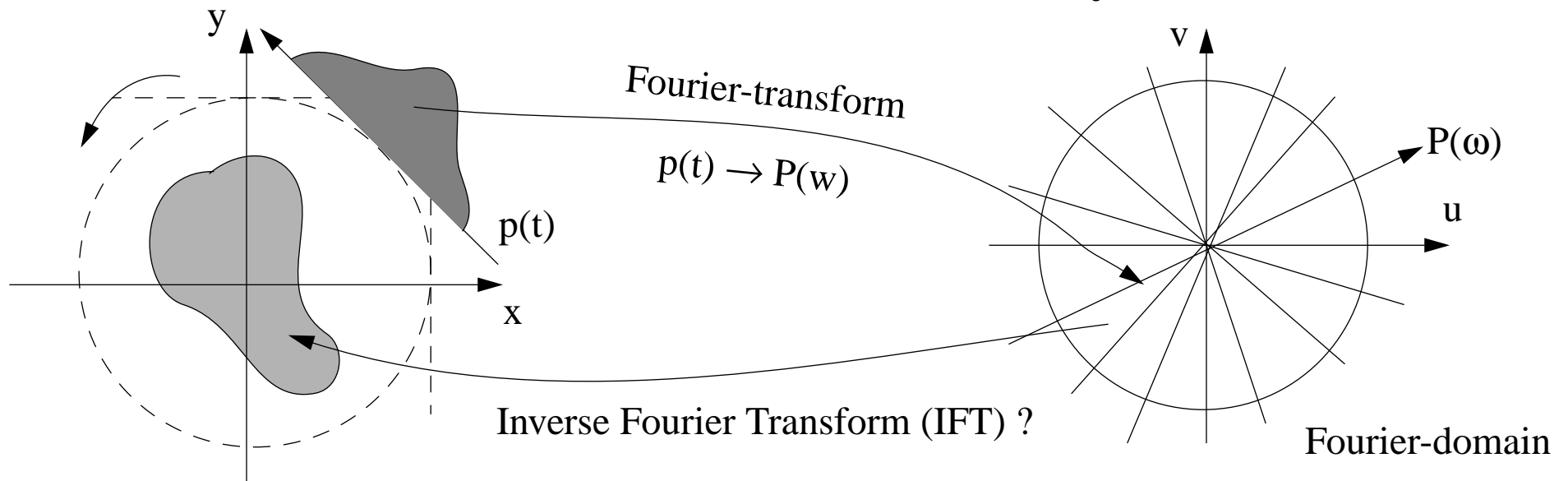


cone-beam

# CT - Filtered Backprojection (1)

Take many projections (>512) around the object

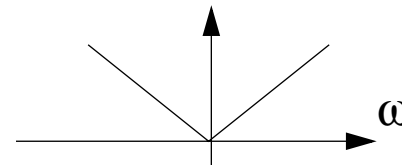
Fourier Slice Theorem: Each Fourier-transformed projection  $P(\omega)$  is a slice in the object's Fourier Transform



- Yes, we could do an IFT, but there are two problems:
  - (1) the outer frequency regions are sampled less densely than the inner regions
  - (2) Fourier transform is on a polar grid -- we want a reconstruction on cartesian grid

• Solution: Filtered Backprojection

(1) multiply each  $P(\omega)$  with a ramp *filter*



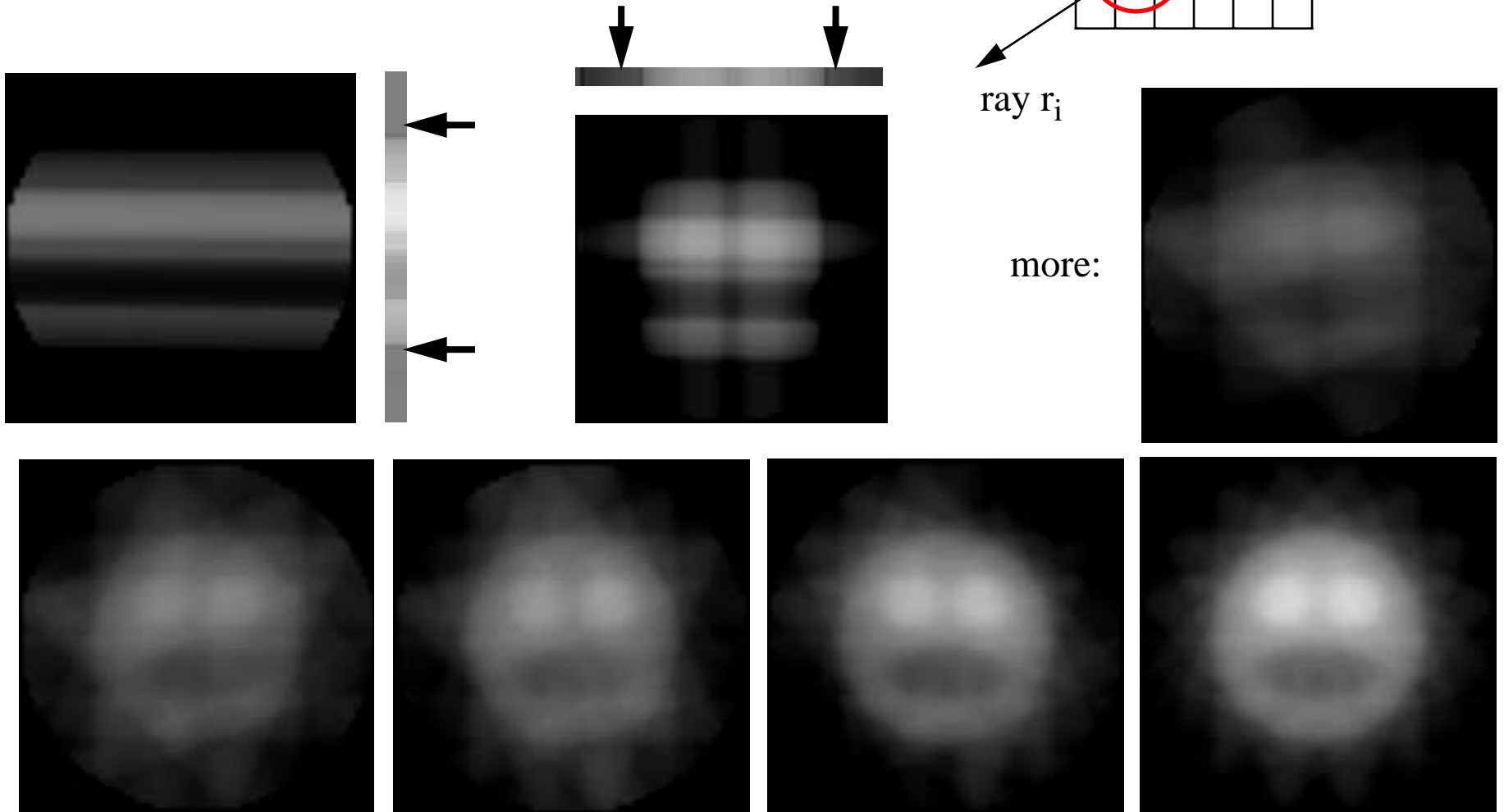
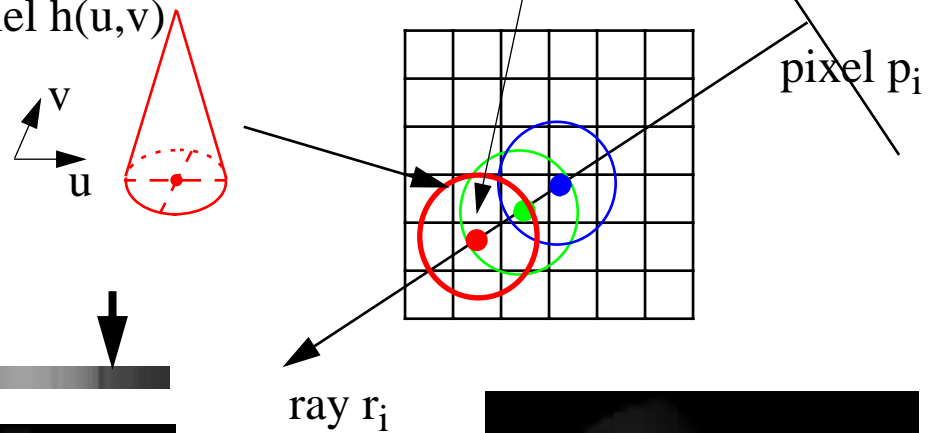
(2) do an IFT on each filtered projection and *backproject* it onto the grid

# CT - Filtered Backprojection (2)

spread ray energy onto the grid voxels, weighted by  $h$

interpolation kernel  $h(u,v)$

“Smear” filtered projection onto the grid via interpolation:  
→ inverse X-ray volume rendering:



# CT - Algebraic Methods

- Alternative to Filtered Backprojection
- Works well when:
  - projections are sparse ( $< 100$ )
  - projections have been acquired at irregular positions
- Popular:
  - ART (Algebraic Reconstruction Technique): one ray at a time
  - SART (Simultaneous ART): one projection at a time

**Algorithm (SART):** Initialize the volume grid (usually zero)

Iteratively, one projection at a time

Project the volume grid

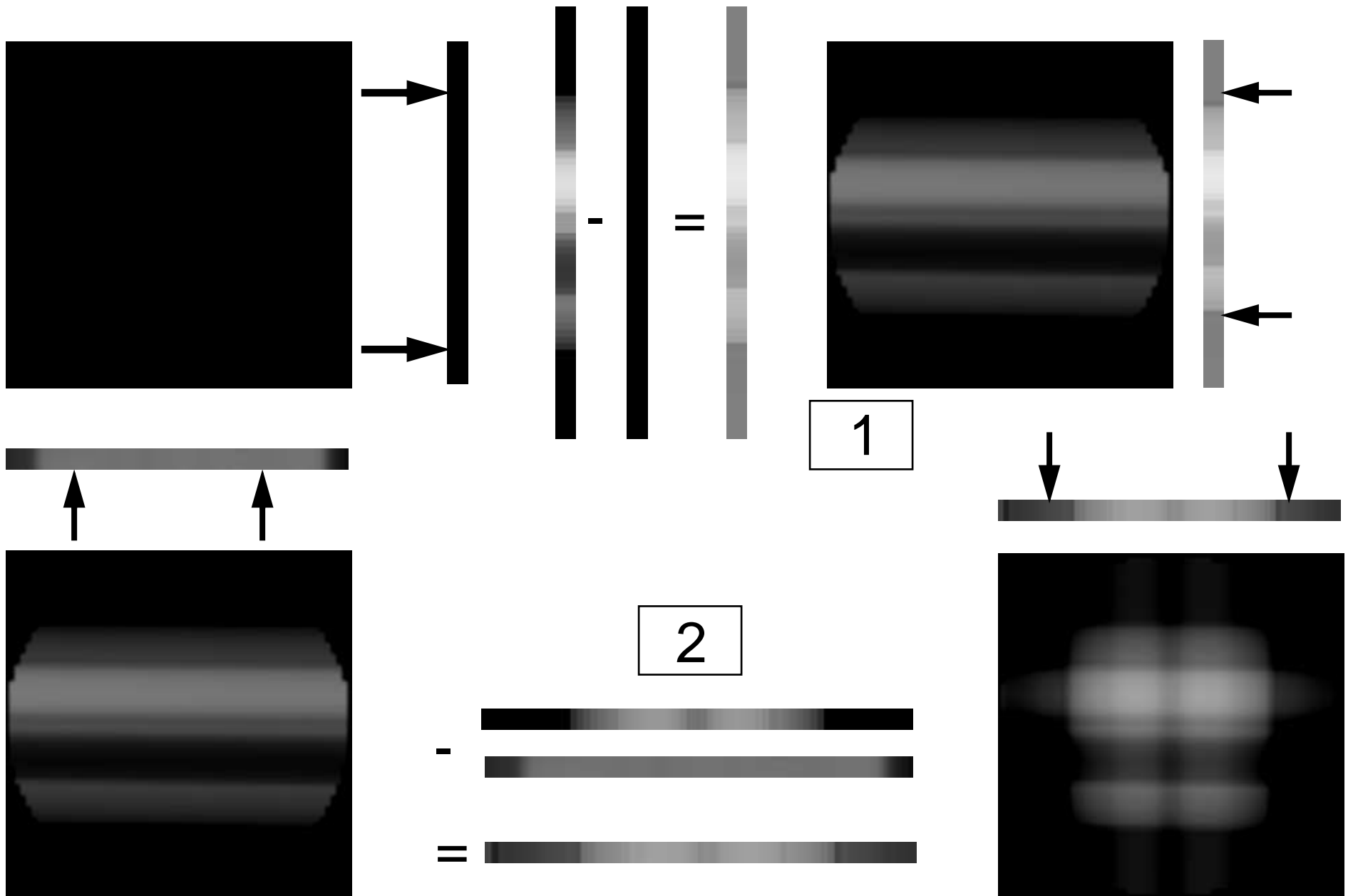
Compute difference (projected image - original image)

Normalize for ray length  $\rightarrow$  correction factors

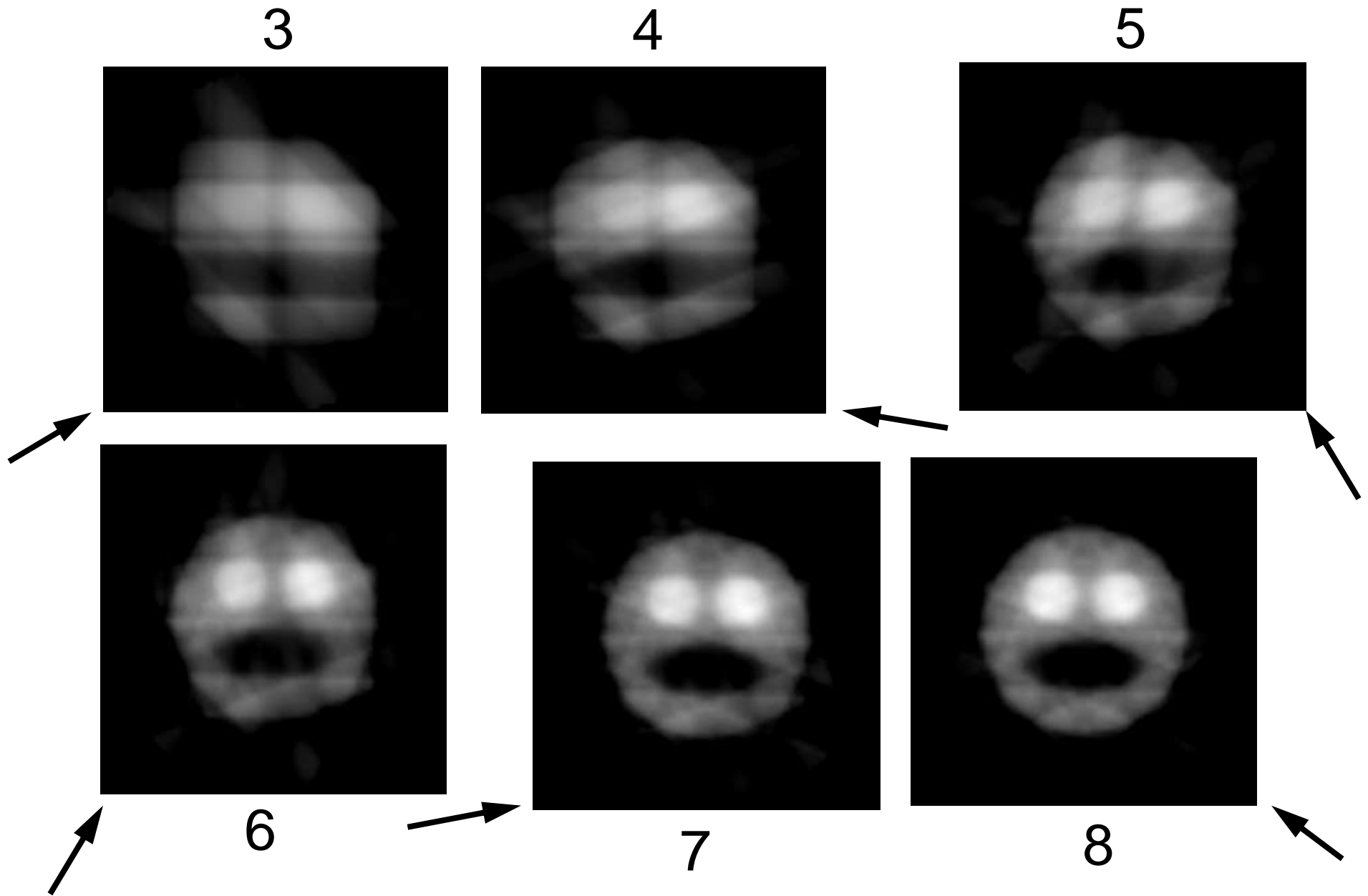
Distribute (backproject) the correction factors onto the grid

do until the grid has converged

# SART - Example (1)



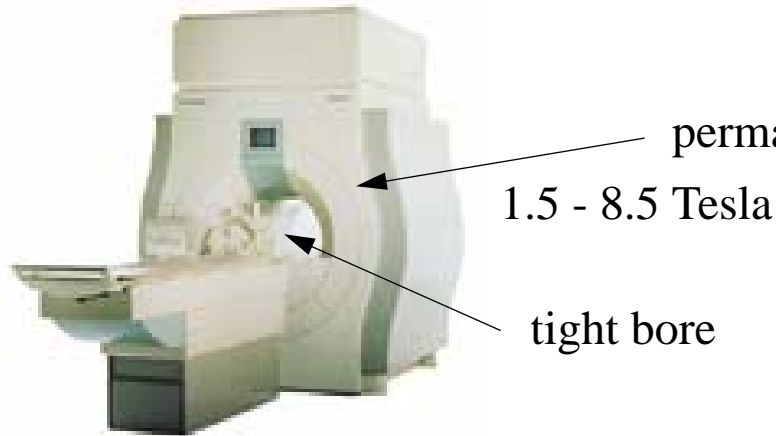
## SART - Example (2)



Note: usually more projections and iterations are necessary (this is a very high-contrast object)

# Magnetic Spin Resonance (MRI) - Overview

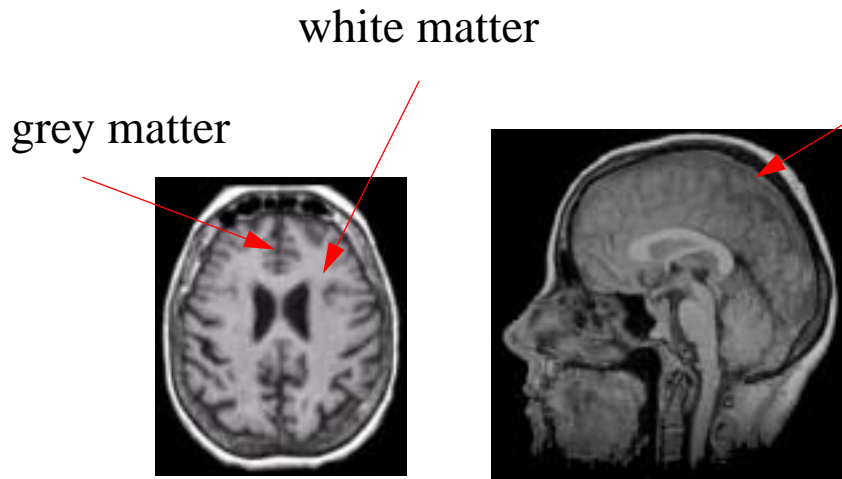
For claustrophobics and MRI-guided surgery:



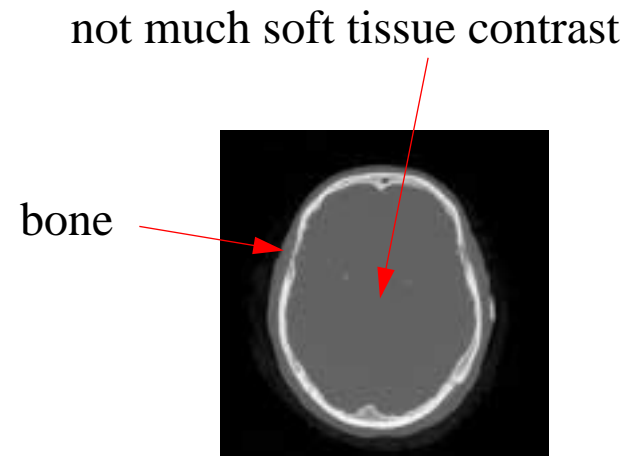
traditional MRI scanner



MRI takes longer (> one breath hold) than CT (< 1s), but shows soft tissue well:



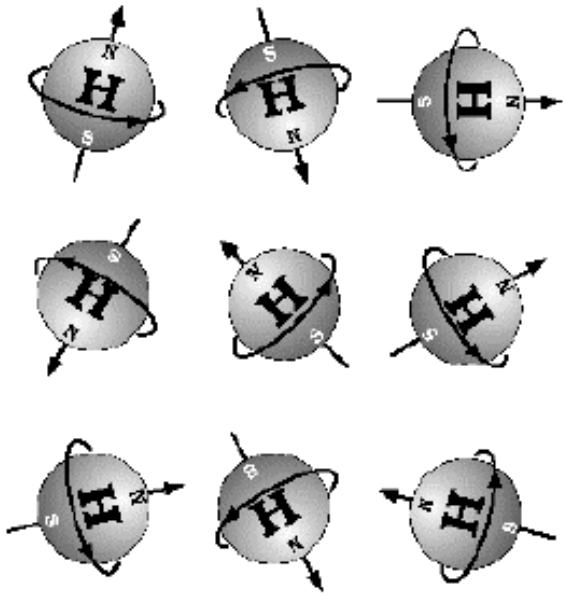
MRI slices



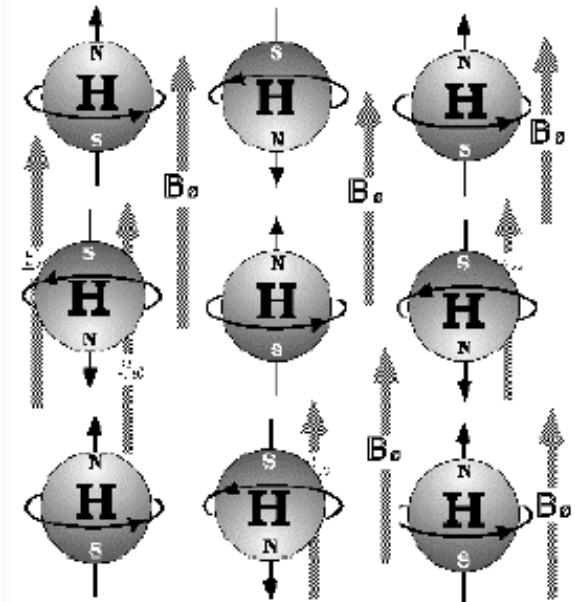
CT brain slice

# Magnetic Spin

- All atoms with an odd number of protons have a magnetic spin
  - example: hydrogen  $H_1$ , which is very prominent in the human body
- The spinning atoms form little magnetic dipoles with N/S pole
- Usually the atom dipoles have random orientations and thus their total magnetic field is zero
- However, if one applies an external magnet field  $B_0$  then the dipoles align themselves with  $B_0$ 
  - they may do parallel or anti-parallel, but parallel will dominate (less energy)



no external field



with external fields  $B_0$

# The Larmor Frequency

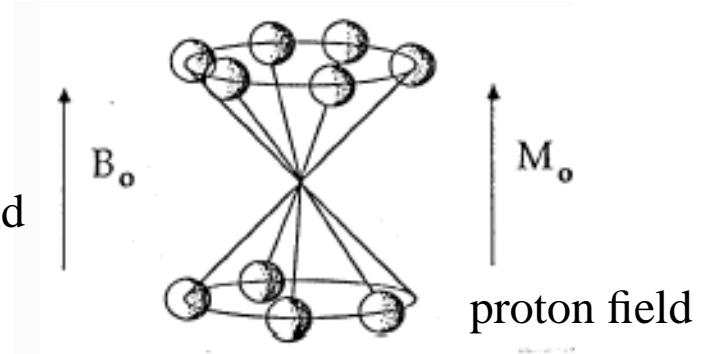
- If there is besides the field  $B_0$  also another field then the protons will spin like a top
- The spin frequency depends on  $B_0$  and the atomic element

$$\omega = B_0 \cdot \gamma$$

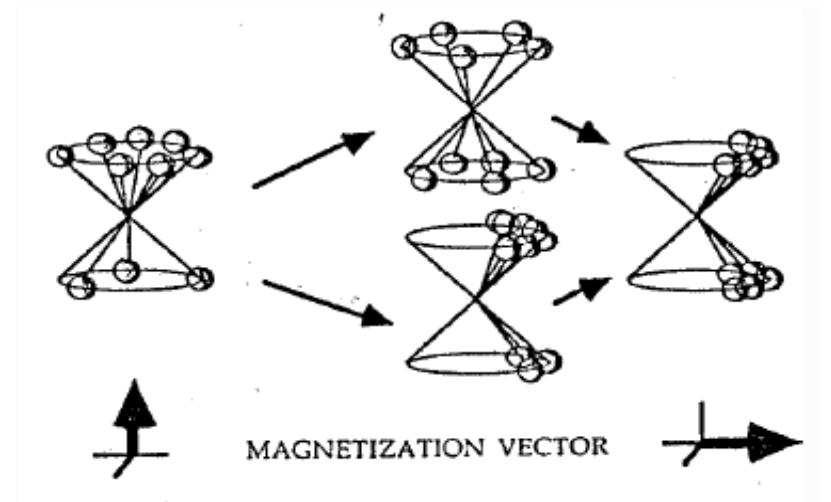
$\omega$ : Larmor frequency

$\gamma$ : gyro-magnetic quotient (for water  $\gamma = 42.577$  MHz/T)

external field

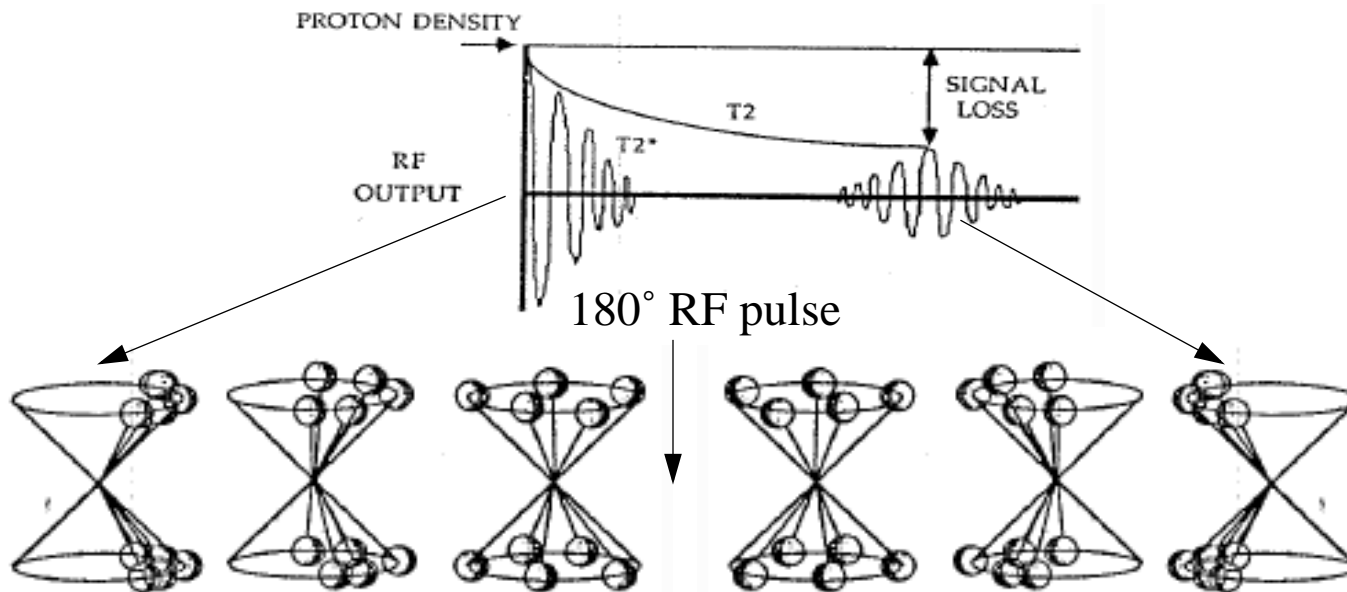


- Important:  $\omega$  depends on the external field  $B_0$
- We would like to measure  $M_0$ , the combined magnetic field of the protons
  - $M_0$  is proportional to the number of protons and thus to the underlying tissue
- How can we do this?
- First: apply external RF field perpendicular ( $90^\circ$ ) to  $B_0$ 
  - this flips the protons down  $90^\circ$
  - it also synchronizes their spin
- Then: turn off the external RF (radio frequency) field



# T2\* Decay and Spin Echo

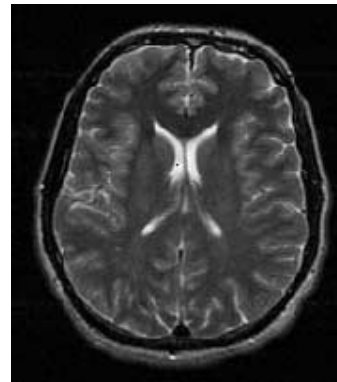
- Introduce an external RF coil (90° angle with  $B_0$ )
  - the spinning protons induce a current into the RF coil which is proportional to  $M_0$
- But two things will happen:
  - the protons get out of sync fast due to the inhomogeneity of the magnetic field  $B_0$  (T2\* decay)
  - they will flip back to their original alignment with  $B_0$
- The former is undesirable and we can fix things by applying a 180° RF pulse at  $t = TE / 2$ 
  - this reverses the spin and after  $t = TE$  all protons will be in sync again (*spin-echo*)



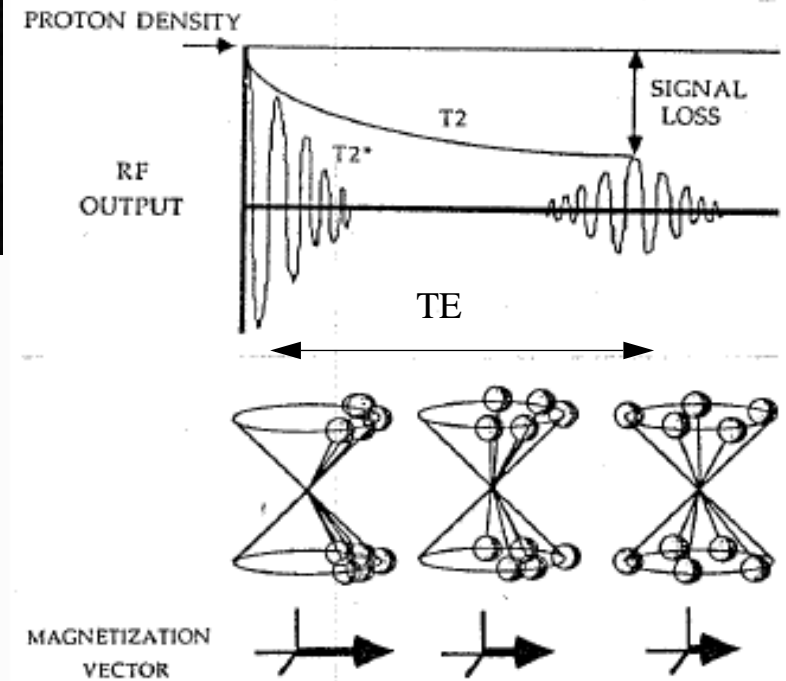
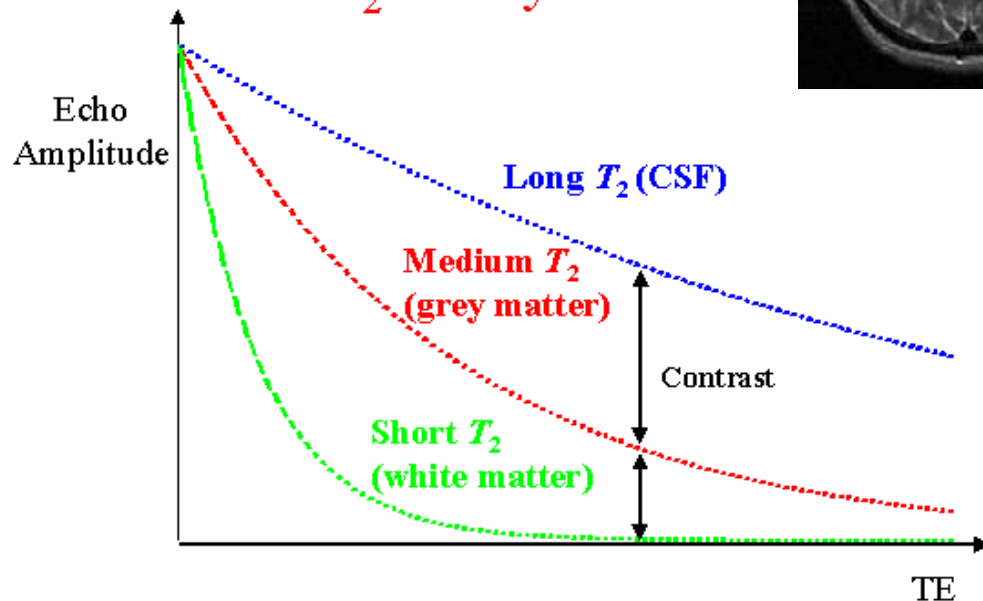
# T2 Relaxation

- BUT: there will still be some dephasing and the measured signal will be less than  $M_0$ 
  - this reduction is due to tissue characteristics and this is what we *want* to measure
  - the received signal is dependent on the T2 time constant of the local tissue
  - the MRI image that results from this measurement is called T2-weighted

Measured signal:  $I = c \cdot e^{-\frac{TE}{T_2}}$



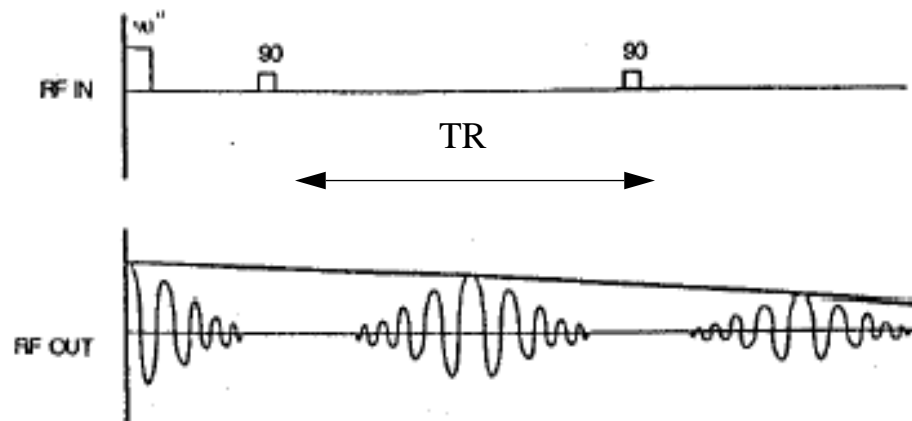
## T<sub>2</sub> Decay Curves



# T1 Relaxation (1)

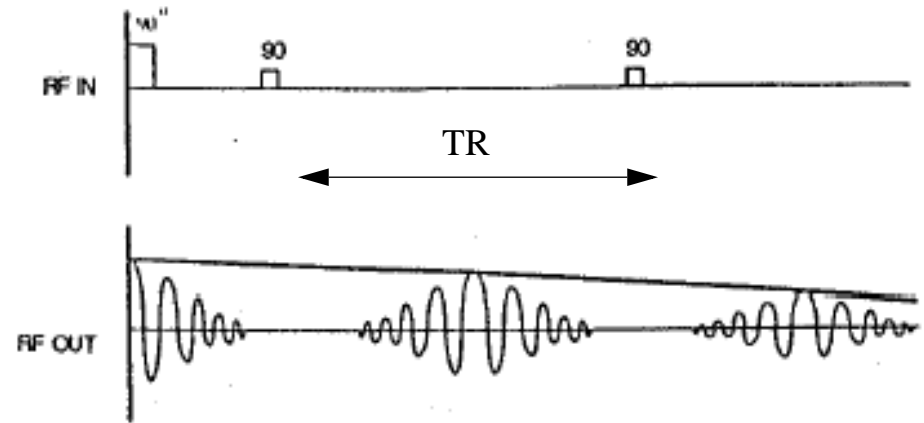
- The protons will flip back to their alignment with  $B_0$ 
  - but the time required for this depends on the tissue characteristics
  - this relaxation behavior is determined by the tissue's time constant T1
- We can measure T1 as well:
- A  $90^\circ$  RF pulse flips the  $B_0$ -aligned component of  $M_0$
- If the proton has not returned fully to  $B_0$  yet, then the flipped  $M_0$  component will be smaller
  - the signal measured at  $90^\circ$  will be smaller as well
  - thus the signal measured is dependent on T1
- The MRI image that results from this measurement is called T1-weighted

$$\text{Measured signal: } I = c \cdot \left( 1 - e^{-\frac{TR}{T1}} \right)$$

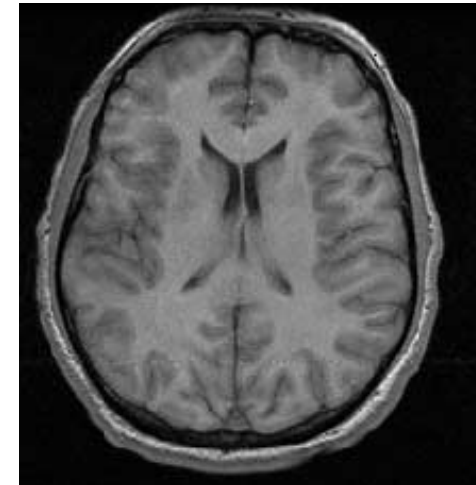
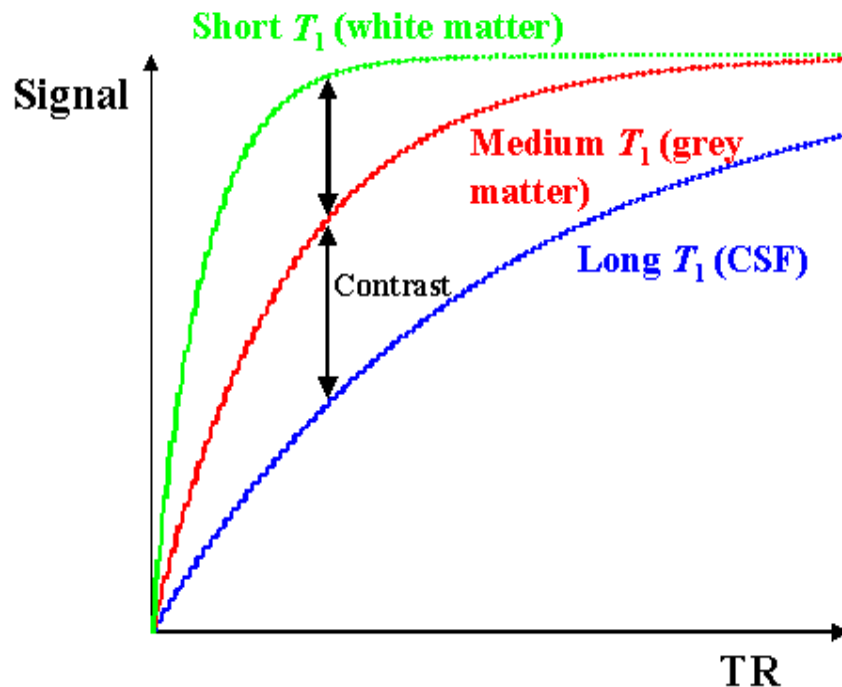


## T1 Relaxation (2)

$$\text{Measured signal: } I = c \cdot \left( 1 - e^{-\frac{TR}{T_1}} \right)$$



### $T_1$ Recovery Curves

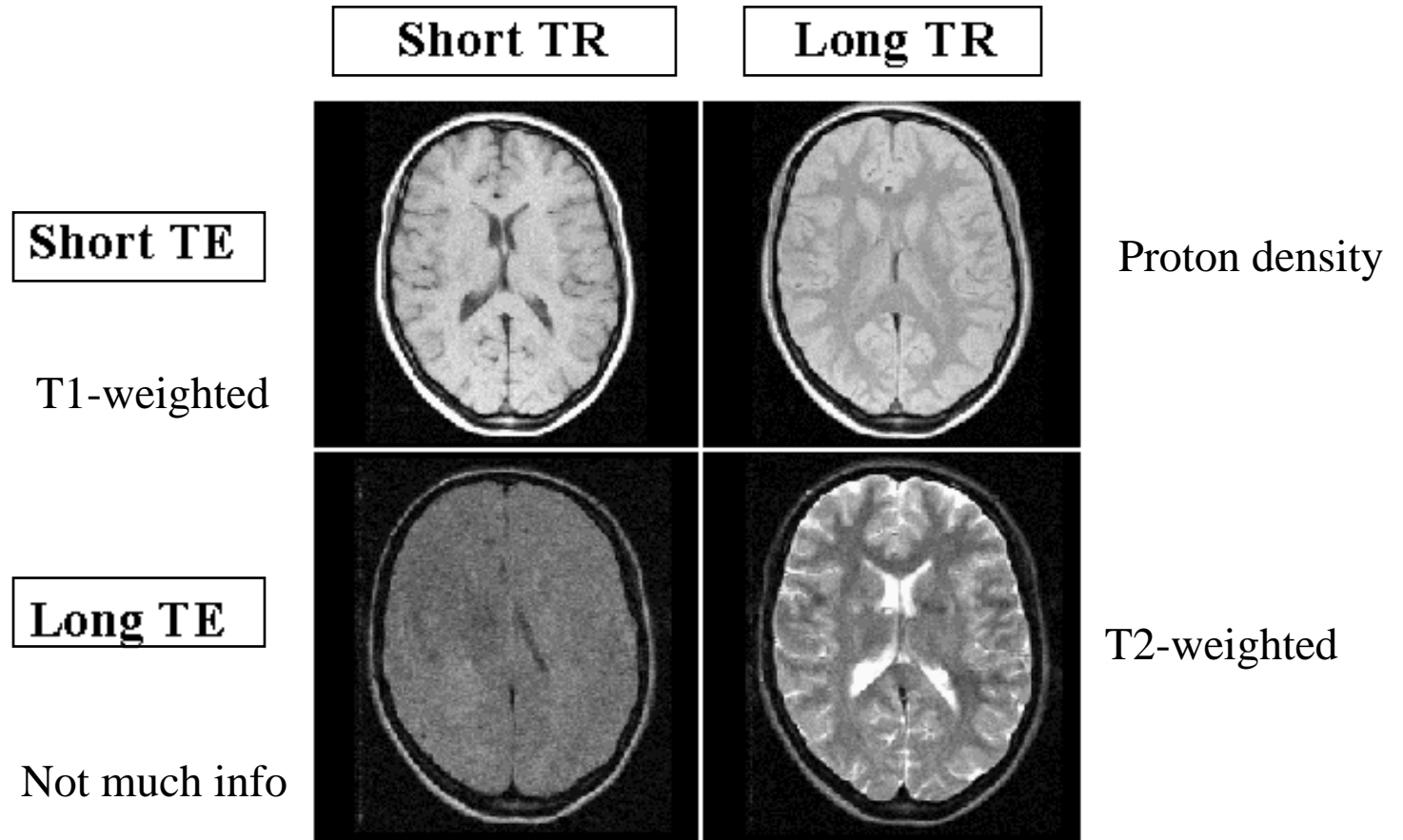


CSF (Cerebral Spinal Fluid) does not show  
Grey matter shows a bit better  
White matter shows best

# MRI - Combined Effects

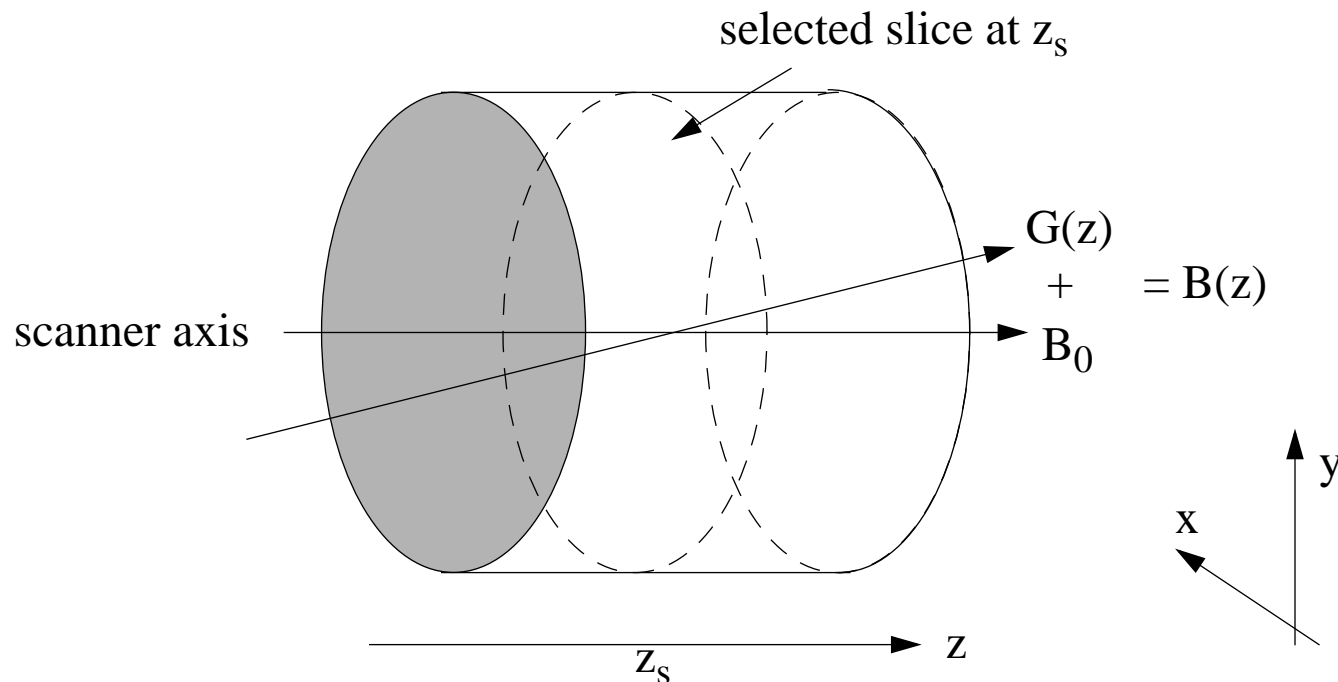
$$\text{Total measured signal: } I = \rho \cdot e^{-\frac{TE}{T2}} \cdot \left( 1 - e^{-\frac{TR}{T1}} \right)$$

$\rho$ : proton density



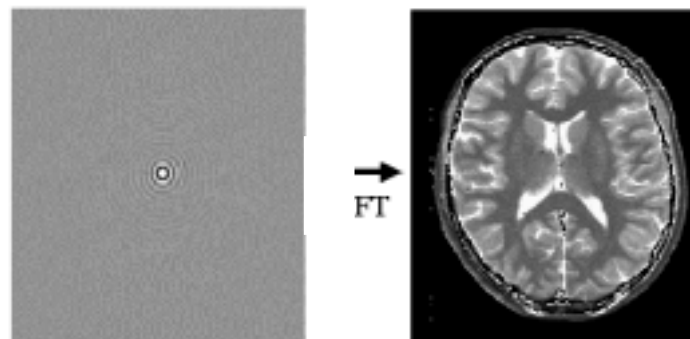
# MRI - Reconstruction (1)

- A measured signal is due to all voxels in the 3D object
- We need to somehow separate the signals coming from the individual voxels
- Recall that the Larmor frequency is a function of  $B_0$ 
  - apply a gradient field such that  $B(z) = B_0 + G_z \cdot z$
  - then tune the external flip RF field to  $\omega(z_s) = B(z_s) \cdot \gamma$  to pick the desired slice at  $z_s$
  - only the protons spinning at  $\omega(z_s)$  will be flipped



## MRI - Reconstruction (2)

- Now only the voxels in slice  $z_s$  will return a combined signal
- Need to distinguish the individual voxels in the  $(x, y)$  slice at  $z_s$
- Prior to signal detection with the RF receiver coil:
  - apply a gradient  $G_x$  (frequency encoding) along  $x$  and  $G_y$  (phase encoding) along  $y$
  - now voxels along  $x$  spin at a different Larmor frequency, along  $y$  at different phase angles
- Measure the signal repeatedly, each time changing the magnitude for the phase encoding
- We get  $N_y$  signals of  $N_x$  data points each (the raw data): Do a 2D Fourier transform of this
  - each sample point in the transform is due to a slice voxel
  - the amplitude of the sample point is the voxel's MRI density in the 2D slice selected by  $G_z$



raw data

after 2D Fourier transform