Algorithm Seminar: warm up session for art-gallery problem

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This discussion session was more of an warm up session for the family of art gallery problems. We mostly discussed the similar flavor problem on graphs. There are two sets of problems which has that flavor. First one is widely known vertex cover (VC) and another is Dominating Set (DS).

We are more concerned with planar graph here, as it is more relevant in the context of art gallery problem. In vertex cover our goal is to cover all the edges where as in dominating set out goal is to find a subset of vertices such that all the vertices are neighbor of some vertex in that subset. So size of dominating set is less than the size of vertex cover.

In figure 1 , it is shown that the bound $\frac{3n}{4}$ is sometimes necessary.

Then we discussed about the lower bound on the set of DS and VC. If we consider planar graph with n vertices then vertex cover size is $\frac{3n}{4}$. This proof is simple as we can have a consistent coloring of a planar graph with 4 colors (according to then famous 4 color theorem). Then the largest number of vertices in a color class has to be at least $\frac{n}{4}$. If we cover all other vertices then we can cover all the edges. Interestingly there is no *easy* algorithm to achieve that bound. So the questions asked are as following,

- Is there an easy $O(n)$ algorithm to achieve $\frac{3n}{4}$ bound ?
- Is there an easy algorithm for *minimal* vertex cover ?

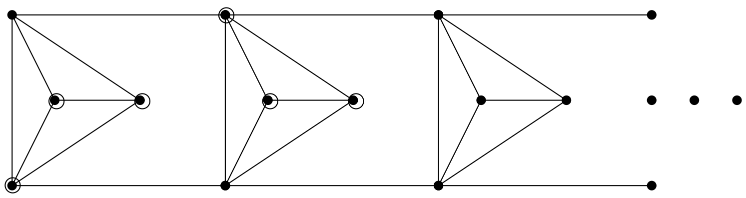

Figure 1: Example of $\frac{3n}{4}$ size vertex cover

There is also an interesting algorithm discussed at the end of the session, in context of DS. If number of edges in a graph increases, how does it affect size of DS. Let the function $f(n, m)$ where n is the number of nodes and m is the number of edges denote the size of DS, what is the characteristics of f if we change m. It is obvious that if $m < n - 1$ then its a disconnected graph, and if $m = \binom{n}{2}$, then size of DS is 1. But we are not certain of its behavior in the midway.

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In this session we did not make much progress in terms of the problems proposed in last section. We proposed more questions on art gallery problem for simple polygons.

If the boundary of P consists of two or more cycles, then P is called a polygon with holes. Otherwise, P is called a simple polygon or a polygon without holes. Two points u and v in a polygon P are said to be visible if the line segment joining u and v lies inside P. So the question asked is what is the minimum number of guards required for guarding a polygon P? By using *triangulation* we can have an obvious approximation as follows,

Theorem 1. *A simple polygon P of n vertices needs at most $\lfloor \frac{n}{3} \rfloor$ guards.*

This bound can be achieved as follows, vertices of P can be coloured using three colours (say, $\{1, 2, 3\}$) such that two vertices joined by an edge of P or by a diagonal in the triangulation of P receive different colours. Although Prof. Mitchell suggested the he wants an efficient triangulation, he also extended the problem for mobile guards.

Returning to the discussion of putting a bound on the vertex cover set of planar graph, an easy bound of $\frac{4n}{5}$ can be achieved by 5 coloring, as five coloring of a planar graph in linear time.

Coming back to the combinatorial expression for the size of DS (Dominating Set) $f(n, m)$ where n is the number of nodes and m is the number of edges, what is the characteristics of f if we change m. It is evident that we can get $f(n, m) = 1$, as long as there is a node with degree $n - 1$. So Prof. Chowdhury proposed an randomized algorithm as follows,

Problem 2. Let G be a complete graph, and we are deleting edges from the graph uniformly, what is the expected number of deletions before we get a graph with, maximum degree of a node is less than $n - 1$.

As we go on deleting vertices the probability of getting a graph with a particular degree sequence changes. Prof. Chowdhury suggests if we delete repeatedly without deleting edges from a particular $(n-1)$ degree node than we can put a bound on the value of expected number of deletion. The exact numerical calculation is yet to be done.