CSE 150 Mini HW 1

Are the sets $\{0, 1, 2\}$ and $\{2, 1, 1, 2, 6 - 6\}$ equal?

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Yes. Order and repetition don't matter, and $6 - 6 = 0$.

Are the sets $\{0, 1, 2\}$ and $\{2, 1, \{0\}\}$ equal?

Are the sets $\{0, 1, 2\}$ and $\{2, 1, \{0\}\}$ equal?

No. $0 \neq \{0\}$

Are the sets $\{dog, cat, mouse\}$ and $\{\{dog, cat, mouse\}\}$ equal?

Are the sets $\{dog, cat, mouse\}$ and $\{\{dog, cat, mouse\}\}$ equal? No. The first set has 3 elements: dog, cat, and mouse. The second set has 1 element: the set $\{dog, cat, mouse\}$.

Are the sets $\mathbb{Z}$ and $\{-x | x \in \mathbb{Z}\}$ equal?

Are the sets $\mathbb{Z}$ and $\{-x|x \in \mathbb{Z}\}$ equal?

Yes. The negative of every integer is an integer, and every integer has a negative.

Is $5 \in \{5, 6, 7\}$?

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Yes.

Is $5 \in \{\{5, 6, 7\}\}$?

Is $5 \in \{\{5, 6, 7\}\}$?

No. The only element of the set is another set, $\{5, 6, 7\}$.

If $a = 0$, is $a \in \emptyset$?

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No. Nothing is in the empty set.

Is $\emptyset \in \emptyset$?

Is $\emptyset \in \emptyset$?

No. Nothing is in the empty set.

Is $\emptyset \subseteq \emptyset$?

Is $\emptyset \subseteq \emptyset$?

Yes.

Is $\emptyset \subset \emptyset$?

Is $\emptyset \subset \emptyset$?

No. If $A \subset B$ then $A \neq B$.

How many integers are there in the set $\{10, 11, green, \{3\}, (7, 8)\}$?

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2. The only integers in this set are 10 and 11. green is obviously not an integer. $\{3\}$ is a set, not an integer. $(7, 8)$ is an ordered pair, not an integer.

Is $\{8, 9\} \subseteq \{\{5, \{8, 9\}\}\}$?

Is $\{8, 9\} \subseteq \{\{5, \{8, 9\}\}\}$?

No. The set $\{8, 9\}$ is an element of $\{\{5, \{8, 9\}\}\}$, not a subset of it.

Is $\{8, 9\} \subseteq \{5, 8, 9\}$?

Is $\{8,9\} \subseteq \{5,8,9\}$?

Yes. Every element of the set $\{8,9\}$ is also an element of the set $\{5,8,9\}$.

$$|\{1, 1, 1, green, 1\}| =$$

$|\{1, 1, 1, green, 1\}| = 2.$

The set has two elements: 1 and green.

If $a = 5$ and $b = 7 - 2$, then $|\{a, b\}| =$

If $a = 5$ and $b = 7 - 2$, then $|\{a, b\}| = 1$.
The set has one element: 5

$$|\{x \in \mathbb{N} \mid 2|x, 7|x, 1 \leq x \leq 10\}| =$$

$|\{x \in \mathbb{N} \mid 2|x, 7|x, 1 \leq x \leq 10\}| = 0.$
There are no natural numbers between 1 and 10 that are divisable by 2 and 7.

If $A \subset B$ and $|B| = 5$, how small or large could A be?

If $A \subset B$ and $|B| = 5$, how small or large could A be? $0 \leq |A| \leq 4$.

Suppose a, b, and c are non-zero integers. You do not know which, if any, are equal. How small or large could the set $\{a, b, c, a - a\}$ be?

Suppose a, b, and c are non-zero integers. You do not know which, if any, are equal. How small or large could the set $\{a, b, c, a - a\}$ be? Note that the set always contains $a - a = 0$. Even if $a = b = c$, we know that $a \neq 0$, so the set must contain at least two distinct elements: a and 0. If a, b and c are all different, then the set could contain as many as 4 elements.

$\{0, 2, 4\} \cup \{1, 3\} =$

$$\{0, 2, 4\} \cup \{1, 3\} = \{0, 1, 2, 3, 4\}$$

$$\{red, blue, green\} \cup \{green, 1, 3\} =$$

$$\{red, blue, green\} \cup \{green, 1, 3\} = \{red, blue, green, 1, 3\}$$

$$\{1, 2\} \cup \{\} =$$

$$\{1,2\} \cup \{\} = \{1,2\}$$

$\emptyset \cup \{red, green\} =$

$$\emptyset \cup \{red, green\} = \{red, green\}$$

$\mathbb{Q} \cup \mathbb{Z} =$

$\mathbb{Q} \cup \mathbb{Z} = \mathbb{Q}$, since $\mathbb{Z} \subseteq \mathbb{Q}$.

$\bigcup_{i=0}^{5}\{2^i, 2^i+1, \ldots, 2^{i+1}-1\} =$

$$\bigcup_{i=0}^{5}\{2^i, 2^i+1, \ldots, 2^{i+1}-1\} = \{1, 2, \ldots, 63\}$$

$\bigcup_{i=0}^{\infty}[-2^{-i}, 10 - 2^{i}] =$

$$\bigcup_{i=0}^{\infty}[-2^{-i}, 10 - 2^{i}] = [-1, 10)$$

$$\left|\bigcup_{i=0}^{5}\{\text{binary strings of length } i \text{ with exactly } \lfloor i/2 \rfloor \text{ zeros}\}\right| =$$

$|\bigcup_{i=0}^{5}\{\text{binary strings of length } i \text{ with exactly } \lfloor i/2 \rfloor \text{ zeros}\}| =$

There is 1 string of length 0 with 0 zeros: the empty string.
There is 1 string of length 1 with 0 zeros: "1".
There are 2 strings of length 2 with 1 zero: "01" and "10".
There are 3 strings of length 3 with 1 zero: "011", "101", "110".
There are 6 strings of length 4 with 2 zeros: "0011", "0101", "0110", "1001", "1010", "1100".
There are 10 strings of length 5 with 2 zeros: "00111", "01011", "01101", "01110", "10011", "10101", "10110", "11001", "11010", "11100".
Thus there are $1 + 1 + 2 + 3 + 6 + 10 = 23$ elements in the union.

$$\{0, 2, 4, 6\} \cap \{2, 6, 8, 10\} =$$

$$\{0, 2, 4, 6\} \cap \{2, 6, 8, 10\} = \{2, 6\}$$

$$\{1, 2, 3\} \cap \{\{1, 2, 3\}\} =$$

$$\{1,2,3\} \cap \{\{1,2,3\}\} = \emptyset$$

$\mathbb{Z} \cap \mathbb{Q} \cap \mathbb{N} =$

$$\mathbb{Z} \cap \mathbb{Q} \cap \mathbb{N} = \mathbb{N}$$

$$\{2k|k \in \mathbb{N}\} \cap \{3x|x \in \mathbb{N}\} =$$

$$\{2k|k \in \mathbb{N}\} \cap \{3x|x \in \mathbb{N}\} = \{6k|k \in \mathbb{Z}\}$$

$\{-x|x \in \mathbb{N}\} \cap \mathbb{N} =$

$$\{-x | x \in \mathbb{N}\} \cap \mathbb{N} = \{0\}$$

$$(\mathbb{Z} \times \mathbb{Q}) \cap (\mathbb{Q} \times \mathbb{Z}) =$$

$$(\mathbb{Z} \times \mathbb{Q}) \cap (\mathbb{Q} \times \mathbb{Z}) = \mathbb{Z} \times \mathbb{Z}$$

$$(\mathbb{Z} \times \mathbb{Q}) \cup (\mathbb{Q} \times \mathbb{Z}) =$$

$$(\mathbb{Z} \times \mathbb{Q}) \cup (\mathbb{Q} \times \mathbb{Z}) = \{(a, b) \in \mathbb{Q} \times \mathbb{Q} | a \in \mathbb{Z} \text{ or } b \in \mathbb{Z}\}$$

$$P(\{1,2,3\}) \cap P(\{2,3,4\}) =$$

$$P(\{1,2,3\}) \cap P(\{2,3,4\}) = \{\emptyset, \{2\}, \{3\}, \{2,3\}\}$$

$P(S) \cap P(T) =$

$P(S) \cap P(T) = P(S \cap T)$.

Proof First we prove that $P(S) \cap P(T) \subseteq P(S \cap T)$. So we need to show that every element of $P(S) \cap P(T)$ is also an element of $P(S \cap T)$. Well, suppose A is an element of $P(S) \cap P(T)$. Then, by definition of intersection and power set, $A \subseteq S$ and $A \subseteq T$, and hence $A \subseteq S \cap T$. Consequently, $A \in P(S \cap T)$. This establishes that $P(S) \cap P(T) \subseteq P(S \cap T)$.

Now we argue that $P(S \cap T) \subseteq P(S) \cap P(T)$. In this case, we must show that every element of $P(S \cap T)$ is also an element of $P(S) \cap P(T)$. Consider any element A of $P(S \cap T)$. By definition of the power set, $A \subseteq S \cap T$, and hence $A \subseteq S$ and $A \subseteq T$. This implies that $A \in P(S)$ and $A \in P(T)$. This implies that $A \in P(S) \cap P(T)$. Thus we have established that $P(S \cap T) \subseteq P(S) \cap P(T)$.

Since $P(S \cap T) \subseteq P(S) \cap P(T)$ and $P(S) \cap P(T) \subseteq P(S \cap T)$, we must have that $P(S) \cap P(T) = P(S \cap T)$.