

CSE 150 Mini HW 2

For sets A and B , $A \setminus B =$

For sets A and B , $A \setminus B = \{x \in A \mid x \notin B\}$

$$\{1, 2, 3\} \setminus \{2, 3, 4\} =$$

$$\{1, 2, 3\} \setminus \{2, 3, 4\} = \{1\}$$

$$\{1, 2, 3\} \setminus \{4, 5, 6\} =$$

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$$\{1, 2, 3\} \setminus \{1, 2, 3\} =$$

$$\{1, 2, 3\} \setminus \{1, 2, 3\} = \emptyset$$

$$\{1, 2, 3\} \setminus \emptyset =$$

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$$\{3k|k \in \mathbb{Z}\} \setminus \{3k|k \in \mathbb{Z}, k \text{ odd}\} =$$

$$\{3k|k \in \mathbb{Z}\} \setminus \{3k|k \in \mathbb{Z}, k \text{ odd}\} = \{6k|k \in \mathbb{Z}\}$$

If $A \cap B = \emptyset$, then $A \setminus B =$

If $A \cap B = \emptyset$, then $A \setminus B = A$

Give an example A and B such that $P(A \setminus B) \neq P(A) \setminus P(B)$.

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Let $A = \{1, 2\}$ and $B = \{2\}$. Then

$$P(A \setminus B) = P(\{1\}) = \{\emptyset, \{1\}\}$$

but

$$\begin{aligned} P(A) \setminus P(B) &= \{\emptyset, \{1\}, \{2\}, \{1, 2\}\} \setminus \{\emptyset, \{2\}\} \\ &= \{\{1\}, \{1, 2\}\} \end{aligned}$$

Recall that set complement, S^C , is relative to some universe that you must infer from context.

$$\{2k | k \in \mathbb{Z}\}^C =$$

$$\{2k|k \in \mathbb{Z}\}^C = \{2k + 1|k \in \mathbb{Z}\}$$

The universe is \mathbb{Z} .

$$\{1/n \mid n \in \mathbb{Z}\}^C =$$

$$\{1/n \mid n \in \mathbb{Z}\}^C = \{a/b \mid a \neq b \wedge a, b \in \mathbb{Z}\}$$

The universe is \mathbb{Q} .

$$\{S \mid S \subseteq A \wedge S \neq A\}^C =$$

$$\{S \mid S \subseteq A \wedge S \neq A\}^C = \{A\}$$

The universe is $P(A)$.

The complement of the set of all programs that terminate is...

The complement of the set of all programs that terminate is...
the set of all programs that do not terminate.
The universe is the set of all programs.

$$P(\{1, 2, 3\}) =$$

$$P(\{1, 2, 3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

Is $1 \in P(\{1, 2, 3\})$?

Is $1 \in P(\{1, 2, 3\})$? No.

If $A \subseteq B$, is $P(A) \subseteq P(B)$?

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Yes. Suppose $X \in P(A)$. This implies that $X \subseteq A$. Since $A \subseteq B$, we must have $X \subseteq B$. Thus $X \in P(B)$.

Is $P(A \times B) = P(A) \times P(B)$?

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No, not even close. For example, if $A = B = \{1\}$, then

$$P(A \times B) = P(\{(1, 1)\}) = \{\emptyset, \{(1, 1)\}\}$$

but

$$\begin{aligned} P(A) \times P(B) &= \{\emptyset, \{1\}\} \times \{\emptyset, \{1\}\} \\ &= \{(\emptyset, \emptyset), (\emptyset, \{1\}), (\{1\}, \emptyset), (\{1\}, \{1\})\} \end{aligned}$$

$$\{1, 2, 3\} \times \{2, 3, 4\} =$$

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$$\{ (1, 2), (1, 3), (1, 4), \\ (2, 2), (2, 3), (2, 4), \\ (3, 2), (3, 3), (3, 4) \}$$

If $A \subseteq B$, is $A \times C \subseteq B \times C$?

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Yes. Suppose $(a, c) \in A \times C$. Then $a \in A$ and $c \in C$. Since $A \subseteq B$, it must be that $a \in B$. Thus $(a, c) \in B \times C$.

Recall that $\{0, 1\}^* = \cup_{i=0}^{\infty} \{0, 1\}^i$, so $\{0, 1\}^*$ contains all binary strings of all finite lengths, e.g. 0000, 1, 01001001, etc. For any string $b \in \{0, 1\}^*$, let $|b|$ be its length, e.g. $|01001| = 5$ and $|00| = 2$. Let b_i be the i th bit of string b , e.g. $0110_0 = 0$, $0110_1 = 1$, $0110_2 = 1$, and $0110_3 = 0$. Consider the function $f : \{0, 1\}^* \rightarrow \mathbb{Z}$, where $f(b) = \sum_{i=0}^{|b|} b_i 2^i$. Is this function an injection?

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No. $f(0) = f(00) = 0$.

What's the difference between $S_1 = (A \times B) \times C$ and $S_2 = A \times (B \times C)$?

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Elements of S_1 are of the form $((a, b), c)$ and elements of S_2 are of the form $(a, (b, c))$. So, the first coordinate of an element of S_1 is an ordered pair, whereas the first coordinate of S_2 is an element of A .

Note: In practice, we don't distinguish between S_1 and S_2 , but instead consider them both to be sets of ordered triples.

We all know that $S = \prod_{i=0}^{\infty} \{0, 1\}$ is uncountable. Now consider $R \subseteq S$ consisting of all such sequences which contain only a finite number of 1s. Prove that R is countable.

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R is obviously infinite. We can view every element of $r \in R$ as a real number between 0 and 1 via

$$q(r) = \sum_{i=0}^{\infty} r_i 2^{-i-1}$$

The function $q : R \rightarrow \mathbb{R}$ is clearly injective. Furthermore, $q(r)$ is rational, so in fact, $q : R \rightarrow \mathbb{Q}$. Since we have found an injection from R to \mathbb{Q} , $|R| \leq |\mathbb{Q}|$.

Comment: R is called a direct sum, or coproduct.