

CSE150 Fall 2007 Midterm Exam

Name: \_\_\_\_\_

- You may not use any reference materials during this exam.
- Electronic devices, including calculators, cell phones, mp3 players, and laptops are all prohibited.
- You may not use your own scratch paper. The exam has plenty and you can ask for more if needed.
- You may not leave the classroom once the exam has been distributed.
- Communicating with other students in any way is prohibited.

**Academic Honesty:** I understand that if I cheat on this exam in any way, I will receive the maximum possible penalty, including an F in this course.

Name (print): \_\_\_\_\_

Signature: \_\_\_\_\_

Problem	Score
1	/25
2	/10
3	/20
4	/10
5	/10
Total	/75

## Sets, Functions, etc.

(10 points) Describe and give the cardinality of each of the following sets.

- $\{1\} \times \{1, 2, 3, 4, 5\}$
- $\mathbb{Z} \times \mathbb{Q} \times \mathbb{N}$
- $\{2k - 1 | k \in \mathbb{N}\} \setminus \{2k + 1 | k \in \mathbb{N}\}$
- $\{(x, y, z, w) \in B^4 | xy + zw < 2\} \Delta \{(x, y, z, w) \in B^4 | x + y + z + w \geq 2\}$
- $\{0, \{0\}, \{\{0\}\}\} \cap \{\{0\}\}$

(15 points) Fill in the blanks in the following table:

$A$	$B$	$f : A \rightarrow B$	$C$	$f(C)$	<i>injective</i>	<i>surjective</i>
$\mathbb{Q}^+$	$\mathbb{R}$	$f(x) = \sqrt{x}$	$\{2k   k \in \mathbb{Z}\}$			
	$\mathbb{Z}$	$f(x) = \lfloor x/3 \rfloor$		$\{2, 3\}$		<i>yes</i>
$\{0, 1, \dots, 7\}$		$f(x) = 2(x \bmod 6)$	$\{2, 7\}$			
$\mathbb{N}$	$\mathbb{Z}$		$\{0, 1, 2, 3, 4, 5\}$	$\{0, 1, -1, 2, -2, 3\}$ ,	<i>yes</i>	<i>yes</i>

## Boolean Logic

(2 points) Write down the truth table for the following function:

$$f(A, B, C) = (B \implies (A \wedge C)) \wedge (\neg B \implies (A \iff \neg C))$$

(8 points) Let  $g(A, B, C)$  be any boolean function, and suppose  $f(A, B, C, D) = (D \wedge g(A, B, C)) \vee (\neg D \wedge g(A, B, C))$ . Prove that  $f(A, B, C, D) = g(A, B, C)$

## Relations

(20 points) Let  $A$  be a set,  $R \subseteq A \times A$ ,  $T : P(A \times A) \rightarrow P(A \times A)$ ,  $S : P(A \times A) \rightarrow P(A \times A)$ ,  $T(R)$  gives the transitive closure of  $R$ , and  $S(R)$  gives the symmetric closure of  $R$ .

- Prove that if  $R$  is symmetric and reflexive, then  $T(R)$  is an equivalence relation.

- Prove that if  $R$  is reflexive and transitive, then  $S(R)$  is an equivalence relation.

## Induction

(10 points) Let  $x_1, x_2, \dots$  be any infinite sequence of integers where  $x_{2k} = 0$  for all  $k \in \mathbb{N}$ . Define  $c_1 = \frac{1}{x_1}$  and, for  $i > 1$ ,  $c_i = \frac{1}{x_i + c_{i-1}}$ . Prove that  $c_i$  is an integer for all even integers  $i$ .

## Number Theory

(10 points) Let  $n \in \mathbb{Z}$ ,  $S = \{0, 1, \dots, n-1\}$ ,  $a \in S$ , and  $f : S \rightarrow S$  be defined by  $f(x) = ax \pmod{n}$ . Prove that  $f$  is a bijection if  $n$  is prime. When  $n$  is not a prime, give an  $a$  such that  $f$  is not a bijection.