

CSE 150: Problem Set #1

Due: Sep. 26, 2007

Problem 1

Write down the following sets, and their cardinalities (sizes):

Note: If you don't know the proper notation for something, just describe it as well as you can.

1. $\{x \in \mathbb{Z} \mid |x| < 10\} \setminus \mathbb{N}$
2. $\{\{6, 3, 24\}, \{\emptyset\}, \{\text{forty-blue}\}\} \Delta \{\emptyset, \{6\}\}$
3. $P(\{5, \{6, 7, \{8\}\}, 9\})$
4. $\bigcap_{i=0}^{10} \{x \in \mathbb{N} \mid x \leq i\}$ (*Note: this is the “summation” notation for intersection*)
5. $(S \times S) \times S$, where $S = \{3, 4\}$
6. $P(P(P(\emptyset)))$
7. $P(P(P(\emptyset) \setminus \{\emptyset\}) \setminus \emptyset) \setminus \{\emptyset\}$

Problem 2

Indicate whether each relation R is reflexive, symmetric, or transitive:

1. $S = \{1, 2, 3, \dots, 10\}$,
 $R \subseteq S \times S$,
 $R =$ “is less than the square of”
2. $R \subseteq \mathbb{R} \times \mathbb{R}$,
 $(x, y) \in R \Leftrightarrow y = \sin x$
3. $R \subseteq \text{People} \times \text{People}$,
 $R =$ “is the sibling of”

Problem 3

Recall that for a function f , $f(X) = \{y \mid y = f(x), x \in X\}$ and $f^{-1}(Y) = \{x \mid f(x) \in Y\}$. For each of the following claims, either prove it is true or provide a counterexample. If it is not true of all functions, determine if it is true for surjections, injections, or bijections, and prove that it is.

1. $f(f^{-1}(Y)) = Y$
2. $f^{-1}(f(X)) = X$
3. $f(f^{-1}(f(X))) = f(X)$
4. $f^{-1}(f(f^{-1}(Y))) = f^{-1}(Y)$

5. $f(X \cup Y) = f(X) \cup f(Y)$
6. $f(X \cap Y) = f(X) \cap f(Y)$
7. $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$
8. $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$

Problem 4

Let $n \geq 2$ be an integer, and define a relation on the integers by $a \equiv b$ if n divides $a - b$. Prove that \equiv is an equivalence relation. (Def: n divides x iff there exists an integer k such that $x = kn$)