

CSE 150: Problem Set #3

Problem 1

Construct an array of 7 integers such that, when quicksort runs on the array, every call to partition splits its input array exactly in half.

Now suppose you have two arrays, A and B , each of length n . Suppose also that, when quicksort runs on A , each call to partition splits its input array exactly in half, and likewise for B . Construct a new array C of size $2n + 1$ such that, when quicksort runs on C , each call to partition splits its input array in half.

Problem 2

Solve the following recurrence relations (in all cases, $T(1) = 0$). You only need to find $\Theta()$ solutions, not exact solutions.

- $T(n) = T(\lfloor n/2 \rfloor) + 1$
- $T(n) = 2T(\lfloor n/2 \rfloor) + 1$
- $T(n) = 4T(\lfloor n/2 \rfloor) + 1$
- $T(n) = T(n - 1) + 1$
- $T(n) = 2T(n - 1) + 1$
- $T(n) = T(\lfloor n/2 \rfloor) + n$
- $T(n) = 2T(\lfloor n/2 \rfloor) + n$ (yeah, I know we did one like this in class)
- $T(n) = 4T(\lfloor n/2 \rfloor) + n$

Problem 3

Pick three of the recurrences from Problem 2 and carefully prove, via induction, that your answer is correct. For convenience, you may assume that n is a power of two.

Problem 4

Use the partition algorithm to write a function **find-kth-smallest**(A , n , k) that, given an array, A , of n integers, returns the k th smallest integer in the array. In other words, your algorithm should return the integer that would be in position k after the array is sorted. A trivial solution to this problem is

```
procedure find-kth-smallest( $A$ ,  $n$ ,  $k$ )
  qsort( $A$ ,  $n$ )
  return  $A[k]$ 
```

This algorithm would have running time $O(n \log n)$. Your algorithm should be faster.

Example tabbing environment

In case you decide to write your homework in latex, here's an example of using the tabbing environment to typeset pseudo-code using the commands defined in the source of this latex file.

```
procedure find-kth-smallest(A, n, k)
  qsort(A, n)
  if foo
    hello
    for blah blah blah
      hello
  else
    goodbye
  for blah blah blah
    hello
  return A[k]
```