1 Don't forget the little o

Let f and g be monotonic functions, with $f(n) = o(g(n))$.

a. Can you always find a function *between* f and g? Find h such that $f(n) = o(h(n))$ and $h(n) = o(g(n))$.

b. Recall that for a monotonically increasing function m, if $a < b$, then $m(a) < m(b)$; and logarithms are monotonic. Is $\log f(n) = o(\log g(n))$ for all f and g as described above?

2 Ordering functions by asymptotic bound

Let k be a fixed positive integer. Put the following functions in increasing asymptotic order. Identify which functions, if any, are asymptotically equivalent.

$$
\begin{aligned}
f_1(n) &= \log n^{\log n} \\
f_2(n) &= 5^{\sqrt{n}} \\
f_3(n) &= \sum_{i=0}^{k} n^i \\
f_4(n) &= n^{k+1} \\
f_5(n) &= \log n^{\log \log n} \\
f_6(n) &= 2^n \\
f_7(n) &= \sum_{i=0}^{n} i^k \\
f_8(n) &= \binom{n}{k}
\end{aligned}
$$

3 Recurrence relation system

You have been given a system of difference equations:

$$
\begin{aligned}
f_n &= -f_{n-1} + g_{n-1} + h_{n-1}, & f_0 &= 4 \\
g_n &= 2f_{n-1} + g_{n-1} - h_{n-1}, & g_0 &= 1 \\
h_n &= f_{n-1} + g_{n-1} - h_{n-1}, & h_0 &= 1
\end{aligned}
$$

a. This system was created from a recurrence relation:

$$f_n = af_{n-1} + bf_{n-2} + cf_{n-3}$$

Find a, b, and c.

b. Find an integer root of the characteristic equation:

$$x^3 - ax^2 - bx - c = 0$$

c. In fact, the sequence f_n is a linear combination of the Fibonacci sequence and the sequence of the powers of the integer root. Find, by inspection, the general term f_k.