

CSE150 Fall 2008 Midterm Exam

Name: \_\_\_\_\_

- You may not use any reference materials during this exam.
- Electronic devices, including calculators, cell phones, mp3 players, and laptops are all prohibited.
- You may not use your own scratch paper. The exam has plenty and you can ask for more if needed.
- You may not leave the classroom once the exam has been distributed.
- Communicating with other students in any way is prohibited.

**Academic Honesty:** I understand that if I cheat on this exam in any way, I will receive the maximum possible penalty, including an F in this course.

Name (print): \_\_\_\_\_

Signature: \_\_\_\_\_

Problem	Score
1	/25
2	/15
3	/20
4	/15
5	/15
Bonus?	/15
Total	/105

## Sets, Functions, etc.

(25 points) Fill in the blanks in the following table:

$A$	$B$	$f : A \rightarrow B$	$C$	$f(C)$	<i>injective</i>	<i>surjective</i>
$\mathbb{Q}$	$\mathbb{Z}$	$f\left(\frac{a}{b}\right) = b$ $\gcd(a, b) = 1$ $b > 0$	$\{2k \mid k \in \mathbb{Z}\}$			
	$\mathbb{R}$	$f(x, y) = x \lfloor y \rfloor$		$\{2, 3\}$		<i>yes</i>
$\mathbb{Z} \times \mathbb{Z}$		$f(a, b)$ $= (a + b) \bmod 7$	$\{2, 7\}$			
$\mathbb{Z}$	$\mathbb{Z}$		$\{0, 1, 2, 3, 4, 5\}$	$\{1, 2, 0, 4, 5, 3\}$	<i>yes</i>	<i>yes</i>

## Boolean Logic

(5 points) Write down the truth table for the following function:

$$f(A, B, C) = (B \implies (A \vee C)) \wedge (\neg B \implies (A \wedge \neg C))$$

(10 points) Prove that  $a \wedge (a \vee b) = a$  and that  $a \vee (a \wedge b) = a$ .



## Induction

(15 points) Prove that for all  $k \in \mathbb{N}$ , there exists an  $N$  such that for all  $n > N$ ,  $\log^k n < n$ .

## Number Theory

(15 points) For  $n \in \mathbb{N}$ ,  $n > 1$ , let  $I([x]_n) = \{-y \mid y \in [x]_n\}$ . Prove that  $I([x]_n) = [-x]_n$ .

## Sorting (Bonus?)

(15 points) Write down the partition algorithm and state its properties.