CSE 150 Fall 2008: Problem Set #1

Note: If you don't know the proper notation for something, just describe it as well as you can.

Problem 1

Write down the truth tables for the following expressions.

1. $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$
2. $P \wedge (Q \Leftrightarrow R)$

Problem 2

Define $P\#Q = \neg(P \vee Q)$. Prove that every boolean expression can be written using just #. (Hint: reduce $\neg$ and $\vee$ to #.)

Problem 3

Each of the following statements is false. Prove it.

1. $\exists x \in \mathbb{R}.\forall y \in \mathbb{R}.xy = 1$
2. $\forall p \in \mathbb{N}.p$ is prime $\Rightarrow 2^p - 1$ is prime
3. $\exists x \in \{0, 1\}.(x + 1)^2 \neq_2 (x^2 + 1)$

Problem 4

Indicate whether each relation R is reflexive, symmetric, or transitive:

1. $S = \{1, 2, 3, \ldots 10\}$,
   $R \subseteq S \times S$,
   $R =$ "is less than the square of"
2. $R \subseteq \mathbb{R} \times \mathbb{R}$,
   $(x, y) \in R \Leftrightarrow y = \sin x$
3. $R \subseteq$ People $\times$ People,
   $R =$ "is the sibling of"

Problem 5

Recall that for a function $f : A \to B$ and $X \subseteq A$ and $Y \subseteq B$, $f(X) = \{y|y = f(x), x \in X\}$ and $f^{-1}(Y) = \{x|f(x) \in Y\}$. For each of the following claims, either prove it is true for every function f and sets $X_1, X_2 \subseteq A$, $Y_1, Y_2 \subseteq B$, or provide a counterexample. If it is not true of all functions, determine if it is true for surjections, injections, or bijections, and prove that it is.

1. $f(X_1 \cup X_2) = f(X_1) \cup f(X_2)$
2. $f(X_1 \cap X_2) = f(X_1) \cap f(X_2)$
3. $f^{-1}(Y_1 \cup Y_2) = f^{-1}(Y_1) \cup f^{-1}(Y_2)$
4. $f^{-1}(Y_1 \cap Y_2) = f^{-1}(Y_1) \cap f^{-1}(Y_2)$