CSE 150: Problem Set #2

Problem 1

Describe and count the equivalence classes of each relation given below. Is each equivalence class the same size?

1. $R \subseteq \mathbb{Z} \times \mathbb{Z}$. xRy iff $x \bmod 7 = y \bmod 7$
2. $R \subseteq \mathbb{Z} \times \mathbb{Z}$. xRy iff $\lfloor \frac{x}{7} \rfloor = \lfloor \frac{y}{7} \rfloor$. (Definition: $\lfloor z \rfloor$ is the largest integer less than or equal to z)
3. Let $B = \{0, 1\}$, and B^n be the set of n-bit strings. $R \subseteq B^n \times B^n$. xRy iff x and y have the same number of 0s.

Problem 2

Let $R \subseteq A \times B$ be a relation on A and B. Let

$$\mathcal{T}(R) = \{R' \subseteq A \times B | R \subseteq R' \text{ and } R' \text{ is transitive}\}$$

Let

$$T(R) = \bigcap_{R' \in \mathcal{T}(R)} R'$$

Prove that

$$T(R) = \{(a, b) | \exists x_1, \ldots, x_n \text{ such that } aRx_1Rx_2R \ldots Rx_nRb\}$$

Problem 3

Is there a set X such that $P(X)$ is countablly infinite? Prove your answer.

Problem 4

The Fibonacci sequence is defined as follows: $F_1 = F_2 = 1$, and for $n > 2$, $F_n = F_{n-1} + F_{n-2}$. What is $F_n \bmod 2$? Prove your answer by induction.

Problem 5

Consider the following game. The game begins with several piles of tokens between the players, and each player has an unlimited supply of tokens. The players take turns and, on their turn, a player can take 1 token from a pile, say pile i, and may place any number of tokens on the piles to the left of pile i. The winner is the player that draws the last token. Prove that the game must eventually end.

Errata: There are finitely many piles between the players. Number them 1 through n. A player chooses to draw a token from some non-empty pile i and can place tokens into the piles numbered greater than i.