CSE 150: Problem Set #3

Problem 1

Construct an array of 8 integers such that, when quicksort runs on the array, every call to partition splits its input array exactly in half.

Now suppose you have two arrays, A and B, of length n. Suppose also that, when quicksort runs on A, each call to partition splits its input array exactly in half, and likewise for B. Construct a new array, C, of size $2n$ such that, when quicksort runs on C, each call to partition splits its input array in half.

Problem 2

Solve the following recurrence relations (in all cases, $T(1) = 0$). You only need to find $O()$ solutions, not exact solutions.

- $T(n) = T(\lfloor n/2 \rfloor) + 1$
- $T(n) = 2T(\lfloor n/2 \rfloor) + 1$
- $T(n) = 4T(\lfloor n/2 \rfloor) + 1$
- $T(n) = T(n-1) + 1$
- $T(n) = 2T(n-1) + 1$
- $T(n) = T(\lfloor n/2 \rfloor) + n$
- $T(n) = 2T(\lfloor n/2 \rfloor) + n$ (yeah, I know we did one like this in class)
- $T(n) = 4T(\lfloor n/2 \rfloor) + n$

Problem 3

Pick three of the recurrences from Problem 2 and carefully prove, via induction, that your answer is correct. For convenience, you may assume that n is a power of two.

Problem 4

You have been given n seemingly identical coins. In fact, however, one of these coins is counterfeit, and has a slightly different weight than the other coins, which are all genuine and weigh the same.

You want to identify which coin is counterfeit. To do this, you have been given a scale that allows you, in one weighing, to compare two sets of k coins, with the scale favoring the left side, the right side, or balanced.

For parts (a) and (b), assume that you know that the counterfeit coin is *lighter* than a genuine coin.

(a) What is the least number of weighings, w, that you know you will need, in the worst case, to identify the counterfeit coin out of n coins?

(b) Give a strategy to identify the counterfeit coin in w weighings.

If you do not know whether the counterfeit coin is actually *lighter* or *heavier,* you can easily find out with one weighing, but, in fact, that is rather inefficient. We explore this in parts (c) and (d).

(c) What is the least number of weighings, w', that you know you will need, in the worst case, to identify the counterfeit coin out of n coins and to determine whether it is *lighter* or *heavier*? Show that for some n, $w' = w$.

(d) Give a strategy to identify, in 3 weighings, the counterfeit coin out of 13 coins. (Hint: determine in advance, for all 3 weighings, which coins to use.)