

# CSE 150: Problem Set #7

December 11, 2008

## Problem 1

Write a card shuffling algorithm that will produce each possible permutation with equal probability.

## Problem 2

- Suppose you have an infinite supply of beads of  $k$  different colors. How many ways can you put  $p$  beads on a string such that the beads are not all of the same color?
- Suppose  $p$  is prime. If you tie the string into a circle, how many different necklaces can you make, if two necklaces are considered equivalent if they are rotations of each other? Note that, as before, necklaces cannot consist of just one color.
- Prove that if  $p$  is prime and  $k \not\equiv 0 \pmod{p}$ , then  $k^{p-1} \equiv 1 \pmod{p}$ . This fact is known as Fermat's Little Theorem.

## Problem 3

Imagine we throw  $n$  balls into  $n$  bins. We then collect all the balls that are in a bin containing more than 1 ball. Suppose there are  $k$  such balls. We now repeat the process with these  $k$  balls, i.e. we throw them into  $k$  new, empty bins, collect the balls that are in bins with more than 1 ball, etc. This process stops when all the balls land in separate bins. How long will it take until this happens, assuming that everything behaves averagely?