

CSE150 Fall 2008 Final Exam

- You may not use any reference materials during this exam.
- Electronic devices, including calculators, cell phones, mp3 players, and laptops are all prohibited.
- You may not use your own scratch paper. The exam has plenty and you can ask for more if needed.
- You may not leave the classroom once the exam has been distributed.
- Communicating with other students in any way is prohibited.

Academic Honesty: I understand that if I cheat on this exam in any way, I will receive the maximum possible penalty, including an F in this course.

Name (print): _____

Signature: _____

Problem	Score
1	/15
2	/20
3	/10
4	/25
5	/10
6	/25
7	/15
Total	/120

1 Recurrence Relations

Solve the following recurrence relations (assuming $T(1) = 0$). You need only give $\Theta(\cdot)$ answers.

- (10 points) $T(n) = T(n/2) + n \log n$

- (5 points) $T(n) = 7T(n/4) + n$

2 Induction Introduction

- (10 points) Prove that $\sum_{i=1}^n i \cdot i! = (n+1)! - 1$

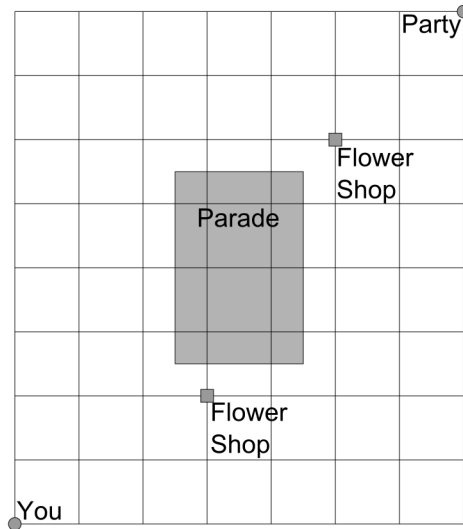
- (10 points) Prove that 21 divides $4^{n+1} + 5^{2n-1}$ for all positive integers n .

3 Monotonic Maps

(10 points) Suppose $A = \{1, \dots, n\}$ and $B = \{1, \dots, m\}$, where $m \geq n$. A function $f : A \rightarrow B$ is increasing if $a < b$ implies that $f(a) < f(b)$. How many increasing functions are there from A to B ?

4 Party Paths

Suppose you are located at $(0, 0)$ in the downtown of a city, and you have to walk 15 blocks to get to $(7, 8)$ for a Halloween party. In particular, in one step, if you are at (i, j) , you could go either to $(i+1, j)$ or $(i, j+1)$.



(a) (5 points) How many different paths are there? (ignore the parade and flower shops for this part of the question)

(b) (10 points) As you are ready to leave for a Christmas party, you realize that you have forgotten to get a gift. A quick *Google Maps* search shows that there are flower shops at $(3, 2)$ and $(5, 6)$. How many different paths will allow you to purchase flowers on the way to the party? (continue to ignore the parade)

(c) (10 points) You are headed to another party on New Year's Eve, but there is a parade in the center of the city, from $(3, 3)$ to $(4, 5)$. How many different paths avoid the parade? (Hint: Split the path into parts.)

5 Glutted Graphs

(10 points) Let G be an undirected graph with n vertices. Suppose every vertex of G has degree at least $\lfloor \frac{n}{2} \rfloor$. Prove that, for every pair of nodes u and v in G , there exists a path of length at most 2 from u to v .

7 Sort-of Sorting

Let A be an $n \times n$ matrix of integers. Imagine you perform the following two operations:

- Sort each column of A .
- Sort each row of A .

(15 points) Prove that the columns of A will still be sorted.