CSE 150 Fall 2009: Homework #2

Note: If you don't know the proper notation for something, just describe it as well as you can.

Hint: Use induction.

Problem 1

The Fibonacci sequence is defined as follows: $F_1 = F_2 = 1$, and for $n > 2$, $F_n = F_{n-1} + F_{n-2}$. What is $F_n \bmod 2$? Prove your answer.

Problem 2

Prove that $\Sigma_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$.

Problem 3

Prove that, if $x \neq 1$, then $\Sigma_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$.

Problem 4

Let $x_1 = \sqrt{1+1}$ and define $x_n = \sqrt{1 + x_{n-1}}$ for all $n > 1$. Prove that x_i is irrational for all i. You may assume that $\sqrt{2}$ is irrational.

Problem 5

Consider a tournament of n teams in which each team plays every other team exactly once. Show that after the tournament completes, we can always arrange the teams in some order $T_1, \ldots, T_n$ such that T_i beat T_{i-1} for all $2 \leq i \leq n$.

Problem 6

Consider the following game. The game begins with several piles of tokens between the players, and each player has an unlimited supply of tokens. The players take turns and, on their turn, a player can take 1 token from a pile, say pile i, and may place any number of tokens on the piles to the left of pile i. The winner is the player that draws the last token. Prove that the game must eventually end.

More precisely, the game begins with n piles of coins. On each turn, the current player selects a non-empty pile i and removes one token from that pile and places tokens on piles $j > i$. Players cannot create new piles.