

CSE150 Fall 2009 Midterm Exam

Name: _____

- You may not use any reference materials during this exam.
- Electronic devices, including calculators, cell phones, mp3 players, and laptops are all prohibited.
- You may not use your own scratch paper. The exam has plenty and you can ask for more if needed.
- You may not leave the classroom once the exam has been distributed.
- Communicating with other students in any way is prohibited.

Academic Honesty: I understand that if I cheat on this exam in any way, I will receive the maximum possible penalty, including an F in this course.

Signature: _____

Problem	Score
1	/20
2	/15
3	/15
4	/25
5	/20
6	/15
Total	/110

Basic Number Theory

(a) (3 points) What is $\phi(23)$?

(b) (3 points) What is $\phi(15)$?

(c) (3 points) What is $7^2 \pmod{15}$?

(d) (3 points) What is $7^{642} \pmod{15}$?

(e) (4 points) What is $7^{-1} \pmod{15}$?

(f) (4 points) Find integers s and t such that $15s + 23t = 1$.

Induction

(15 points) Let $x_0 = 1$ and define $x_n = \frac{1}{x_{n-1}}$. Prove by induction that, for all $k \in \mathbb{N}$, $x_{2k} > \frac{\sqrt{5}-1}{2}$ and $x_{2k+1} < \frac{\sqrt{5}-1}{2}$.

Cardinality

(15 points) Indicate whether each set is finite, countably infinite, or uncountable.

(a) $\mathbb{N} \times \mathbb{Q}$

(b) $\prod_{n=0}^{\infty} \{0, 1\}$

(c) $\{0, 1\}^n$, where $n \in \mathbb{N}$

(d) $\bigcup_{n=0}^{\infty} \{0, 1\}^n$

(e) $\mathbb{Q} \cup \bigcup_{n=0}^{\infty} \{0, 1\}^n$

Analyzing the Euclidean Algorithm

(a) (7 points) Prove that if $b \leq a$, then $a \bmod b < \frac{a}{2}$.

(b) (8 points) Assume computing $a \bmod b$ takes 1 time unit. Let $T(a, b)$ be the amount of time it takes to compute $\gcd(a, b)$, when $b \leq a$. By using the fact that $\gcd(a, b) = \gcd(b, a \bmod b)$, prove that $T(a, b) \leq 1 + T(b, a/2)$.

- (c) (10 points) Assume that for any $a > 0$, we can compute $\gcd(a, 0)$ in 1 time unit. Prove by strong induction that, when $ab \neq 0$, $T(a, b) \leq 2 + \log_2 ab$. (Hint: Remember that $\log_2 \frac{x}{2} = \log_2 x - 1$).

Graph Theory

(15 points) Suppose an odd number of people attend a party. Prove that, no matter who shakes hands with whom, someone at the party must shake hands with an even number of people. (Hint: make a graph of who shakes hands; apply a theorem relating $|E|$ and the node degrees.)