Partial Solutions to Homework 5

CSE 373, Prof. Skiena

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- **Problem 1** For the matching between "watch the movie raising arizona?" and "watch da mets raze arizona?", the matching cost is 12 : MMMMMMDSSMMDSSSMMMD-DDSSMMMMMMMMM.

  For the matching between "this is what happens when I type slow" and "htishisth whaty havpens when ui type fasht", the cost is 14 : SSMMSMMIIMMMMMIMMMSMMM-MMMMMMMISMMMMMMISSSSM.

  For the matching between "leonard skiena", "lynard skynard", the matching cost is 6: MDSMMMMMMMDSMMIIM.

- **8-3.** (a) Let $C(i,j)$ denote the length of the matching substring that ends with ith element of X and jth element of Y. Then the recursion is as follows:

$$C(i,j) = \begin{cases} C(i-1,j-1)+1, & \text{if } X[i] = X[j]; \\ 0, & \text{if } X[i] \neq X[j]; \end{cases} \tag{1}$$

  (b) Simply start from each element in X and search in Y the longest substring that ends or starts from this element.

- **8-7.** (a) $20 = 10*2 = 10+6+1*4 = 10+1*10 = 6*3+1*2 = 6*2+1*8 = 6*1+1*14 = 1*20$, so $C(20) = 7$.

  (b) Let C(n) denote minimal number of coins needed to make change of n, then we have the recursion: $C(n) = \min\{C(n-d_1), C(n-d_2), \cdots C(n-d_k)\}$.

- **8-14.** Omitted

- **8-19.** Simply place as many as possible books, with the fixed order, into the shelf.

- **8-20.** (a) Consider books of height 1, 10, 10. If we place $\{1,10\}$ in one shelf and $\{10\}$ in another, the over all height would be 20. Then it better to put $\{10,10\}$ together.

  (b) The idea is to insert one new book, consider the all the books in the previous shelf to see if the cost can be reduce by inserting this book into the non-empty shelf or by grouping it with some book to a new empty shelf.

- **9-1.** $(x+y+v_1)(\overline{v_1}+\overline{z}+v_2)(\overline{v_2}+w+v_3)(\overline{v_3}+u+\overline{v})(\overline{x}+\overline{y}+v_4)(\overline{v_4}+z+v_5)(\overline{v_5}+\overline{w}+v_6)(\overline{v_6}+u+v)(x+\overline{y}+v_7)(\overline{v_7}+\overline{z}+v_8)(\overline{v_8}+w+v_9)(\overline{v_9}+u+\overline{v})(v_{10}+x+\overline{y})(\overline{v_{10}+x+\overline{y}})$

- **9-2.** Similar in the textbook Figure 9.7.
- **9-8.** For each instance of vertex cover problem $(G = (V, E), k)$, the reduction to the baseball card problem is as follows. We first give each edge $e \in E$ a unique name corresponding the names in the baseball card problem. For each vertex $v \in V$, we put all its neighboring edges into one packet. Then by this way G has a vertex cover of size at most k if and only if the we can collect all the names in at most k packets.
- **9-10.** Reduce from the Clique problem (G, k) to the dense subgraph problem. Let $y = \binom{k}{2}$ and ask if there is a solution for this dense subgraph problem.
- **9-11.** The reduction is as follows: change any clique problem G by adding a new edge and a new node to each node in G, *i.e*, extruding an edge from each node. By this way, if there is a clique of size k in G, there must be an independent set of size k (those k new extruded vertices connected to the k vertices in the clique must be independent). This completes the reduction.
- **9-12.** Simply ask, for a Hamiltonian path problem $H(G)$, if there is a low degree spanning tree of degree at most 2. We can prove that for such a spanning tree there can not be only two nodes with degree 1, and all the other nodes have degree 2. Thus it's equivalent to Hamiltonian path problem.

  If there are three nodes with degree 1, we can find a 'crossroad' that these three leaves meet, which must have degree of at least 3.
- **9-13.** We may reduce from the vertex cover problem G to this problem. The set S is composed of all the vertices in G. For each edge in G, we make a subset of size two that have its two end nodes. Thus, a vertex cover of size at least k corresponding to a hitting set of size at least k.

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