Homework 2 Solution

Problem 4:

Let S[i , j] be the shortest string which is a super-sequence of B_1[1 … i] and B_2[1 … j],

S[0,0] = 0, S[i,0] = i, and S[0,j] = j.

Recurrence:

$$S[\,i\,,j\,] = min \begin{cases} S[\,i-1\,,j-1\,]+1 \quad if\ B1[i] = B2[j] \\ S[\,i-1\,,j\,]+1 \\ S[\,i\,,j-1\,]+1 \end{cases}$$

The runtime is O(nm), where length of B1 = n and length of B2 = m.

Problem 5:

Let P[i , j] be the minimum number of inserts for string S (Si…Sj) to become a palindrome.

When comparing characters, if the case is to add a character at the beginning or at the end, we increase the cost by 1; if the characters match, the cost stays the same.

Recurrence:

$$P[\,i\,,j\,] = min \begin{cases} P[\,i+1\,,j-1]+1 \quad if\ Si = Sj \\ P[\,i\,,j-1\,]+1 \\ P[\,i+1\,,j\,]+1 \end{cases}$$

Run time is $O(n^2)$ where n is the size of the string S.

Problem 6:

Let P[i , j] be the amount of pizza taken that ends with the i^{th} and j^{th} slice.

P[i , j] = 0 if i = j = 0 (no slices left)

P[i , j] = 1 if i = j, I != 0 (1 slice left)

Recurrence:

$$\mathrm{P}[\,\mathrm{i}\,,\mathrm{j}\,] = max \begin{cases} \mathrm{Si} + \sum_{k=i+1}^{j} (S_k - \mathrm{P}[\,\mathrm{i} - 1, \mathrm{j}\,]) \\ \mathrm{Sj} + \sum_{k=i}^{j-1} (S_k - \mathrm{P}[\,\mathrm{i}\,, \mathrm{j} - 1\,]) \end{cases}$$

Run time is $O(n^2)$ for there are n slices.