# CSE526 Spring 2004
# Homework Two
# Problem 3

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# 1 Problem

Using the direct denotational semantics in Sections 2.2-2.4, prove that the equation above equation (2.2) on [Reynolds, p. 28] holds, i.e.,

$$[[\text{while } b \text{ do } c]] \ \sigma = [[\text{if } b \text{ then } (c; \text{while } b \text{ do } c) \text{ else skip}]] \ \sigma$$

In other words, when we have defined the meaning of "while" loops using equation (2.4) on [Reynolds, p. 36], we should be able to prove that the above equation holds, as a way of checking that equation (2.4) has the desired meaning.

# 2 Proof:

$$
\begin{align}
RHS \quad &= \quad \text{if } [[b]] \ \sigma \text{ then } [[c; \ \text{while } b \text{ do } c]] \ \sigma \text{ else } \sigma \tag{1} \\
&= \quad \text{if } [[b]] \ \sigma \text{ then } ([[\text{while } b \text{ do } c]])_{\perp\perp}([[c]] \ \sigma) \text{ else } \sigma \tag{2} \\
&= \quad \text{if } [[b]] \ \sigma \text{ then } (Y_{\Sigma \to \Sigma_\perp} F)_{\perp\perp}([[c]] \ \sigma) \text{ else } \sigma \tag{3}
\end{align}
$$

where $F f \sigma = \text{if } [[b]] \ \sigma \text{ then} f_{\perp\perp}([[c]] \ \sigma) \text{ else } \sigma$.

While for the left-hand-side, we have:

$$
\begin{align}
LHS \quad &= \quad [[\text{while } b \text{ do } c]]\sigma \tag{4} \\
&= \quad Y_{\Sigma \to \Sigma_\perp} F \ \sigma \tag{5}
\end{align}
$$

where, $F f \sigma = \text{if } [[b]] \ \sigma \text{ then} f_{\perp\perp}([[c]] \ \sigma) \text{ else } \sigma$.

Note that we have $Y_D \ F \ \sigma = F \ (Y_D \ F) \ \sigma$, therefore,

$$
\begin{align}
LHS \quad &= \quad F \ (Y_{\Sigma \to \Sigma_\perp} \ F) \ \sigma \tag{6} \\
&= \quad \text{if } [[b]] \ \sigma \text{ then } (Y_{\Sigma \to \Sigma_\perp} \ F)_{\perp\perp}([[c]] \ \sigma) \text{ else } \sigma \tag{7}
\end{align}
$$

where, $F f \sigma = \text{if } [[b]] \ \sigma \text{ then} f_{\perp\perp}([[c]] \ \sigma) \text{ else } \sigma$.
Hence, $RHS = LHS$. Proved.

<div align="right">END OF PROOF</div>