

CSE526: Principles of Programming Languages  
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hw5: shared-variable concurrency, lambda calculus  
version: 5pm, 5mar2004

due: 11mar2004

Answer

16th March 2004

# 1 Problem 1 (10pt)

## 1.1 Problem

Consider adding a new command CU ("conditional update") to the programming language of chapter 8. The syntax of CU is:

$$<comm> ::= CU(<var>, <var>, <intexp>, <var>)$$

Informally, the semantics of CU( $x, x_0, e, \text{flag}$ ) is that it executes the following two atomic steps.

- (1) evaluate the expression  $e$  to an integer  $i$
- (2) if  $x = x_0$  then ( $x := i; \text{flag} := 1$ ) else  $\text{flag} := 0$

Each of these two steps executes atomically. Transitions of other threads may occur between the steps.

Extend the transition semantics of section 8.1 with transition rule(s) for CU. Hint: One issue is where to store the value  $k$  between the two steps. One possible approach is to augment the variety of configurations by introducing an additional kind of command.

## 1.2 Answer without crit

$$\begin{aligned} <comm> &::= CU(<var>, <var>, <intexp>, <var>) \\ &\quad | CU2(<var>, <var>, <intcfm>, <var>) \end{aligned}$$

$$\frac{}{<CU(x, x_0, e, \text{flag}), \sigma> \longrightarrow <CU2(x, x_0, i, \text{flag}), \sigma>} \text{ where } i = [[e]]_{\text{intexp}} \sigma$$

$$\frac{}{<CU2(x, x_0, i, \text{flag}), \sigma> \longrightarrow [\sigma]x : i | \text{flag} : 1} \text{ if } \sigma x = \sigma x_0$$

$$\frac{}{<CU2(x, x_0, i, \text{flag}), \sigma> \longrightarrow [\sigma | \text{flag} : 0]} \text{ if } \sigma x \neq \sigma x_0$$

## 1.3 Answer with crit

$$<comm> ::= CU(<var>, <var>, <intexp>, <var>)$$

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$$\frac{}{<CU(x, x_0, e, \text{flag}), \sigma> \longrightarrow <\text{newvar } k := 0 \text{ in (crit (}k := e\text{); crit (if }x = x_0 \text{ then (}x := k; \text{flag} := 1\text{) else }\text{flag} := 0\text{))}, \sigma>}$$

## 2 Problem 2

### 2.1 10.1 (a) (10pt)

$$\begin{aligned}
 & (\lambda d.dd)(\lambda f.\lambda x.f(fx)) \\
 \rightarrow & ((\lambda f.\lambda x.f(fx))(\lambda f.\lambda x.f(fx))) \\
 \rightarrow & \lambda x.(\lambda f.\lambda x.f(fx))((\lambda f.\lambda x.f(fx))x) \\
 \rightarrow & \lambda x.\lambda x_1.(\lambda f.\lambda x.f(fx))x((\lambda f.\lambda x.f(fx))x x_1) \\
 \rightarrow & \lambda x.\lambda x_1.(\lambda x_2.x(x x_2))((\lambda f.\lambda x.f(fx))x x_1) \\
 \rightarrow & \lambda x.\lambda x_1.x(x((\lambda f.\lambda x.f(fx))x x_1)) \\
 \rightarrow & \lambda x.\lambda x_1.x(x((\lambda x_5.x(x x_5))x_1)) \\
 \rightarrow & \lambda x.\lambda x_1.x(x(x(x x_1))) \\
 \rightarrow & \lambda x.\lambda x_1.x(x(x(x x_1)))
 \end{aligned}$$

### 2.2 10.1 (b) (10pt)

The reduction sequence would stop at the first canonical form:

$$\lambda x.(\lambda f.\lambda x.f(fx))((\lambda f.\lambda x.f(fx))x)$$

### 2.3 10.1 (e) (10pt)

$$\begin{aligned}
 & (\lambda d.dd)(\lambda f.\lambda x.f(fx)) \\
 \lambda d.dd \Rightarrow_E & \lambda d.dd \\
 \lambda f.\lambda x.f(fx) \Rightarrow_E & \lambda f.\lambda x.f(fx) \\
 (\lambda f.\lambda x.f(fx))(\lambda f.\lambda x.f(fx)) \Rightarrow_E & \lambda f.\lambda x.f(fx) \\
 \lambda f.\lambda x.f(fx) \Rightarrow_E & \lambda f.\lambda x.f(fx) \\
 \lambda f.\lambda x.f(fx) \Rightarrow_E & \lambda f.\lambda x.f(fx) \\
 \lambda x.(\lambda f.\lambda x.f(fx))((\lambda f.\lambda x.f(fx))x) \Rightarrow_E & \lambda x.(\lambda f.\lambda x.f(fx))((\lambda f.\lambda x.f(fx))x) \\
 \Rightarrow_E & \lambda x.((\lambda f.\lambda x.f(fx))(\lambda f.\lambda x.f(fx))x) \\
 \Rightarrow_E & \lambda x.((\lambda f.\lambda x.f(fx))(\lambda f.\lambda x.f(fx))x)
 \end{aligned}$$